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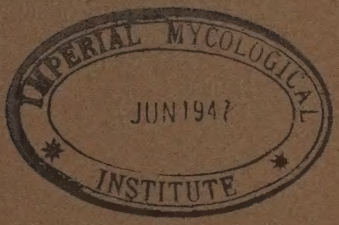

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MATHÉMATIQUES

SUR LES BASES (AU SENS LARGE) DANS LES ESPACES LINÉAIRES

Par A. MARKOUCHEVITCH

(Présenté par S. L. Soboleff, de l'Académie, le 3. VIII. 1943)

1. Les développements classiques en séries, étudiés dans l'analyse, sont les développements en séries potentielles (pour les fonctions analytiques dans un cercle $|z|<r$) et en séries de Fourier (par exemple, pour les fonctions sommables sur le segment $[-\pi,+\pi]$). La différence essentielle entre ces deux types de séries consiste en ce que les puissances $1, z, z^2, \ldots$ présentent une base [au sens de Schauder ([1-2])], nous dirons que c'est une base au sens restreint] pour les fonctions analytiques dans le cercle $|z|<r$*, tandis que les fonctions trigonométriques

$$u_0=\frac{1}{2},\quad u_{2n-1}=\sin nx,\quad u_{2n}=\cos nx\quad (n=1,2,\ldots) \tag{1}$$

ne forment pas une base ni dans l'espace L_1, ni dans l'espace C. Cette dernière circonstance ne diminue pas le rôle des développements de Fourier, et c'est surtout grâce aux propriétés suivantes du système (1):

a) $\{u_n\}$ est un ensemble fondamental dans L_1 ou dans C;

b) il existe une suite de fonctionnelles linéaires $\{L_n\}$ définies dans tout l'espace et formant avec les fonctions (1) un système biorthogonal

$$\left(L_{2n}(f)=\frac{1}{\pi}\int_{-\pi}^{+\pi}\cos nx\,f(x)\,dx,\quad L_{2n+1}(f)=\frac{1}{\pi}\int_{-\pi}^{+\pi}\sin(n+1)\,x\,f(x)\,dx,\right.$$

$$\left.n=0,\ 1,\ 2,\ldots\right);$$

c) de ce que $L_n(f)=0\,(n=0,1,2,\ldots)$ il résulte $f=\theta$ (θ est le zéro de l'espace considéré).

2. Les faits indiqués expliquent la généralisation de la notion de base que nous formulons dans ce qui suit.

Introduisons d'abord la notion d'indépendance linéaire forte ([3]). Soit $\{e_\alpha\}$ un système arbitraire d'éléments (non nécessairement dénombrable) d'un espace E du type F; convenons de le nommer linéairement indépendant au sens fort, en abrégé, système l. i. f., si chaque fois qu'une suite de polynômes par rapport à $\{e_\alpha\}$ converge vers zéro (θ est le zéro de E) il en résulte que la suite des coefficients de chaque u_α fixe converge vers zéro. Chaque système

* La métrique de l'espace de ces fonctions doit être conforme à la convergence au sens de la convergence uniforme dans chaque cercle $|z|\leqslant\rho<r$. Alors l'espace correspondant ne peut pas être un espace de Banach (on considère toutes les fonctions analytiques dans le cercle), mais on peut toujours le construire comme un espace du type F. [Pour les définitions et la terminologie dans l'analyse fonctionnelle voir ([3])].

1. i. f. est formé de fonctions linéairement indépendantes et n'admet pas de développement non trivial du zéro. Mais l'inverse n'est en général pas vrai. Comme exemples servant à expliquer, remarquons que dans l'espace C le système (1) est un système l. i. f., tandis que le système des puissances $1, x, x^2, \ldots$ est formé de fonctions linéairement indépendantes et n'admettant pas de développement non trivial du zéro; en considérant les fonctions d'une variable complexe, analytiques dans le cercle $|z|<r$ et, pour simplifier, continues dans le cercle fermé $|z|\leqslant r$, en prenant pour norme $\|f\| = \max_{|z|\leqslant r} |f(z)|$, on voit que dans ce cas les puissances $1, z, z^2, \ldots$ forment un système l. i. f.

L'importance de la notion d'indépendance linéaire forte découle de ce que chaque base (au sens strict) dans un espace du type F est un système l. i. f. Cependant l'exemple du système (1) dans l'espace C prouve qu'un système fondamental et l. i. f. peut ne pas être une base. Signalons quelques critères nécessaires et suffisants pour l'indépendance linéaire forte d'un système $\{e_\alpha\}$:

α) si une suite de polynômes par rapport à $\{e_\alpha\}$ converge, les suites des coefficients pour chaque e_α convergent aussi;

β) il existe un système de fonctionnelles linéaires $\{L_\alpha\}$ définies sur E et telles que $L_\alpha(e_\beta) = \delta_{\alpha\beta}$, où $\delta_{\alpha\beta} = 0$ pour $\alpha \neq \beta$ et $\delta_{\alpha\alpha} = 1$;

γ) la distance de chacun des éléments de $\{e_\alpha\}$ de la multiplicité linéaire fermée formée par les autres éléments de $\{e_\alpha\}$ est positive.

3. Soit maintenant $\{e_\alpha\}$ un système d'éléments d'un espace E du type F jouissant des propriétés suivantes:

A) $\{e_\alpha\}$ est un ensemble fondamental dans E;

B) $\{e_\alpha\}$ est un système l. i. f.;

C) de ce que toutes les valeurs des fonctionnelles $\{L_\alpha\}$ formant avec $\{e_\alpha\}$ un système biorthogonal (en vertu de la condition B et du critère β du § 2) s'annulent pour un certain $e \in E$, il résulte que $e = 0$.

Nous dirons dans ce cas que $\{e_\alpha\}$ est une base au sens large.

En considérant une base au sens large, on peut associer à chaque élément e un développement formel suivant les éléments $\{e_\alpha\}$:

$$e \sim \sum L_\alpha(e)\, e_\alpha, \tag{2}$$

où $L_\alpha(e)$ sont des nombres réels ou complexes (cela dépend de la nature des nombres qui figurent dans la définition de E comme système linéaire). Si $\{p_n\}$ est une suite quelconque de polynômes par rapport à $\{e_\alpha\}$ qui converge vers e, alors pour chaque e_α la suite des coefficients de ces polynômes converge vers $L_\alpha(e)$. Il en résulte tout d'abord qu'il n'existe qu'un nombre fini ou bien une infinité dénombrable de nombres $L_\alpha(e)$ non nuls (pour un e donné). Ensuite, si une série de la forme $\sum a_\alpha e_\alpha$ converge vers e ou bien contient une suite de sommes partielles qui converge vers e, on a $a_\alpha = L_\alpha(e)$ pour tous les indices α. Enfin, en vertu de la condition B, il correspond à deux éléments e différents deux développements différents du type (2). Donc, si (2) présente une série convergente ou bien seulement contient une suite convergente de sommes partielles, l'élément limite coïncide avec e.

4. Le plus important est le cas d'une base dénombrable. L'existence d'une base (au sens restreint) dénombrable n'est pas démontrée pour les espaces de Banach séparables. Il est d'autant plus intéressant de remarquer qu'il existe dans chaque espace séparable de Banach une base (au sens large) dénombrable.* Pour le voir, le plus simple est de profiter du théorème connu de Banach-Mazur. Soit $\{e_n\}$ un ensemble fondamental d'éléments linéairement indépendants** et soit S un ensemble de l'espace C qui est une image isomorphe et isométrique de E. Soit $\{S_n(x)\}$ l'ensemble des fonctions

* Il existe des espaces du type F qui ne possèdent pas de base au sens large.

** Le cas trivial d'un espace à un nombre fini de dimensions est ici exclu.

continues, $S_n(x) \in S$, qui présentent des images des éléments de $\{e_n\}$; en les orthogonalisant on obtient une suite $\{P_n(x)\}$ de polynômes par rapport à $\{S_n(x)\}$; cette suite est évidemment une base au sens large dans S. L'ensemble des éléments de E dont les images forment l'ensemble $\{P_n(x)\}$ présente un ensemble de polynômes par rapport à $\{e_n\}$; c'est la base au sens large cherchée pour l'espace E.

5. Soit E un espace séparable du type F et $\{e_n\}$ — sa base dénombrable (au sens large). En considérant pour un élément arbitraire $e \in E$ son développement (2), nous pouvons énoncer les trois hypothèses suivantes:

1) La série (2) converge (nécessairement vers e).

2) Elle diverge, mais contient une suite de sommes partielles qui converge (nécessairement vers e).

3) Elle ne contient aucune suite de sommes partielles qui converge.

Ce dernier cas peut réellement avoir lieu même dans l'espace C (pour les séries de Fourier). C'est ce qui a été démontré récemment par M. Menchoff [4]. C'est pourquoi il est intéressant de remarquer que le théorème qu'il a signalé à cette occasion pour les séries de Fourier peut être étendu au cas le plus général que nous considérons ici. A savoir, on peut démontrer* que chaque élément $e \in E$ est la somme de deux éléments e' et e'' (la décomposition $e = e' + e''$ n'est pas unique) tels que pour chacun d'eux le développement correspondant en série suivant les éléments $\{e_n\}$ contient une suite convergente de sommes partielles.

6. Les développements (2) présentent une généralisation très étendue des séries de Fourier; la théorie classique des séries trigonométriques impose donc un grand nombre de questions concernant les développements (2). Parmi ces questions il nous semble que les plus intéressantes sont: le problème des coefficients des séries (2), c'est-à-dire le problème de caractériser les nombres $\{a_n\}$ — coefficients des séries (2), et le problème de la sommation de ces séries par des méthodes quelconques; ces problèmes restent à résoudre.

7. En terminant signalons un théorème tout-à-fait analogue à celui de Boas [5] et qui prouve une continuité *sui generis* de la propriété d'un système d'être une base (au sens large). A savoir, si $\{e_n\}$ est une base (au sens large) pour un espace E du type F et si $\{\varepsilon_n\}$ est une suite d'éléments du même espace vérifiant l'inégalité

$$\left\| \sum L_n(e)(e_n - \varepsilon_n) \right\| \leq \lambda \|e\| \qquad (0 < \lambda < 1) \tag{3}$$

pour chaque $e \in E$, alors $\{\varepsilon_n\}$ est encore une base (au sens large). Dans cette formule λ ne peut pas, en général, être remplacé par l'unité (car dans ce cas il peut arriver que $\{\varepsilon_n\}$ n'est même plus un ensemble fondamental dans E, et aussi il peut ne pas être linéairement indépendant). Remarquons que le théorème de Boas lui-même qu'il a énoncé (sans démonstration) pour les bases au sens restreint et pour les espaces de Banach reste vrai pour les espaces du type F.

Université Lomonossoff.
Moscou.

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* Voir [3], où nous donnons la démonstration pour les espaces de Banach. Remarquons que la condition C du § 3 n'est pas utilisée dans la démonstration.

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THEORY OF ELASTICITY

A CONTACT PROBLEM

By M. Z. NARODETZKY

(Communicated by S. L. Soboleff, Member of the Academy, 22. VII. 1943)

Determine the pressure exercised upon a circular contour of an infinite plate by a solid disc of radius R with force P applied to its centre.

We take the radius of the circular hole of the plate to be equal exactly to the radius of the disc. Denote the total length of the circumference by L, and the arc of contact by L_2. The remaining portion of the circumference, which is under no external force, we denote by L_1, so that $L_1 = L - L_2$. Furthermore, the region occupied by the disc is denoted by S_1, and the region occupied by the infinite plate, by S_2.

Any tangential stress is supposed to be absent on the contact arc L_2, and equality of radial displacements is assumed for points belonging to the disc and the plate.

The problem proposed [1] is reduced to the determination of four functions $\varphi_1(z)$, $\psi_1(z)$; $\varphi_2(z)$, $\psi_2(z)$ of a complex variable $z = x + iy$, which are regular and single-valued in the regions S_1 and S_2, respectively, and satisfy the following boundary conditions:

$$\varphi_1'(t) + \overline{\varphi_1'(t)} - t\varphi_1''(t) - \frac{t^2}{R^2}\psi_1'(t) = \begin{cases} \frac{Pi}{2\pi}\left(\frac{t}{R^2} - \frac{2}{1+\varkappa}\frac{1}{t}\right) \text{ on } L_1, \\ \frac{Pi}{2\pi}\left(\frac{t}{R^2} - \frac{2}{1+\varkappa}\frac{1}{t}\right) + N(t) \text{ on } L_2, \end{cases} \tag{1}$$

$$\varphi_2'(t) + \overline{\varphi_2'(t)} - t\varphi_2''(t) - \frac{t^2}{R^2}\psi_2'(t) = \begin{cases} 0 \text{ on } L_1 \\ N(t) \text{ on } L_2, \end{cases} \tag{2}$$

$$t^2[\bar{t}\varphi_1''(t) + \psi_1'(t)] - \bar{t}^2[t\overline{\varphi_1''(t)} + \overline{\psi_1'(t)}] = \frac{Pi}{2\pi}\frac{1-\varkappa}{1+\varkappa}(t + \bar{t}) \text{ on } L_1, \tag{3}$$

$$t^2[\bar{t}\varphi_2''(t) + \psi_2'(t)] - \bar{t}^2[t\varphi_2''(t) + \overline{\psi_2'(t)}] = 0 \text{ on } L_2, \tag{4}$$

$$t[\varkappa\overline{\varphi(t)} - \bar{t}\varphi'(t) - \psi(t)] + \bar{t}[\varkappa\varphi(t) - t\overline{\varphi'(t)} - \overline{\psi(t)}] =$$

$$= \frac{Pi}{2\pi}(\varkappa \lg R^2 - 1)(t - \bar{t}) \text{ on } L_2. \tag{5}$$

Here t is affix of a point of the contour L, $\varkappa$ is constant of elasticity (the same for the disc and the plate)

$$\varphi(z) = \varphi_1(z) - \varphi_2(z); \quad \psi(z) = \psi_1(z) - \psi_2(z).$$

The first and second conditions represent the well known equations of the theory of elasticity when the external forces are given on the contours of the considered regions. Here $N(t)$ is the unknown real function to be determined.

By the third and fourth equations is expressed the fact that the tangential stresses on the circumference L are equal to zero.

The fifth equation expresses the equality of radial displacements for those points of the disc and plate which lie on the arc of contact L_2.

Multiplying the two members of equation (3) by $\frac{1}{2\pi i}\frac{dt}{t-z}$, where z is an arbitrary point of the region S_1, and integrating over L, we have

$$z\varphi_1'(z)+\frac{z^2}{R^2}\psi_1'(z)=\frac{Pi}{2\pi}\frac{1-\varkappa}{1+\varkappa}\frac{z}{R^2}. \tag{6}$$

Similarly we multiply each of the members of equation (4) by $\frac{1}{2\pi i}\frac{dt}{t-z}$, where z is an arbitrary point of region S_2, and integrate this equation over the contour L to obtain

$$z\varphi_2'(z)+\frac{z^2}{R^2}\psi_2'(z)=a_1'\frac{z}{R^2}+\frac{a_2'}{R^2}-\frac{a_1'}{z}. \tag{7}$$

Here a_1' and a_2' are coefficients of the series obtained by the expansion of the function $\psi_2'(z)$. They are equal to

$$a_1'=\frac{\varkappa Pi}{2\pi(1+\varkappa)}; \quad a_2'=-\frac{R^2}{2\pi i}\int_{L_2}\frac{N(t)}{t}dt,$$

respectively. [The coefficient a_2' is obtained by multiplying equation (2) by $\frac{dt}{t}$ and integrating over L.]

Let us now multiply the two sides of equation (1) by $\frac{1}{2\pi i}\frac{dt}{t-z}$, where z is an arbitrary point of the region S_1, and integrate that equation over L. Taking the relation (6) in consideration, we easily find

$$\varphi_1'(z)=\frac{2a_1'}{\varkappa}\frac{z}{R^2}+\frac{1}{2\pi i}\int_{L_2}\frac{N(t)}{t-z}dt-b_1. \tag{8}$$

Here b_1 is the coefficient of the development of $\varphi_1(z)$ in a Taylor series. Putting $z=0$ in (8), we find

$$b_1=\frac{1}{2\pi i}\int_{L_2}\frac{N(t)}{t}dt.$$

The imaginary part of the coefficient b_1 is assumed to be zero.

Multiplying, further, the equation (2) by $\frac{1}{2\pi i}\frac{dt}{t-z}$, where z is an arbitrary point of region S_2, and integrating over L, we obtain

$$\varphi_2'(z)=-\frac{a_1'}{z}-\frac{1}{2\pi i}\int_{L_2}\frac{N(t)}{t-z}dt. \tag{9}$$

By differentiating the two members of equation (5) with respect to t two times, each time substituting the expression $t\varphi''(t)+\frac{t^2}{R^2}\psi'(t)$ with the known relations from (6) and (7), we, after some simple transformations, bring equation (5) to the form

$$t\varphi''(t)+\overline{t\varphi''(t)}=\frac{a_1'}{R^2}(\bar{t}-t)+2b_1. \tag{10a}$$

We have, further,

$$\varphi_1''(z)=\frac{2a_1'}{\varkappa R^2}+\frac{1}{2\pi i}\int\limits_{L_2}\frac{N(t)}{(t-z)^2}\,dt=\frac{2a_1'}{\varkappa R^2}-\frac{1}{2\pi i}\int\limits_{L_2}N(t)\,d\frac{1}{t-z}=$$

$$=\frac{2a_1'}{\varkappa R^2}-\frac{1}{2\pi i}\frac{N(t)}{t-z}\int\limits_{L_2}+\frac{1}{2\pi i}\int\limits_{L_2}\frac{N'(t)}{t-z}\,dt=\frac{2a_1'}{\varkappa R^2}+\frac{1}{2\pi i}\int\limits_{L_2}\frac{N'(t)}{t-z}\,dt. \tag{10b}$$

In the same manner we get

$$\varphi_2''(z)=\frac{a_1'}{z^2}-\frac{1}{2\pi i}\int\limits_{L_2}\frac{N'(t)}{t-z}\,dt. \tag{10c}$$

We now pass in the equations (10b) and (10c) to the limit, making the point z tend from within and from outside, respectively, to an arbitrary point t of the arc L.

Substituting the obtained values $\varphi''(t)$ and $\overline{\varphi''(t)}$ in the equation (10a), we get:

$$\frac{1}{2\pi i}\int\limits_{L_2}\frac{N'(t_1)}{t_1-t}\,dt_1=\frac{Pi}{4\pi}\left(\frac{1}{t^2}-\frac{1}{R^2}\right)+\frac{b_1}{2t}. \tag{10}$$

Let us consider the function $\eta(z)$ [2] determined by the following equation:

$$\eta(z)=\frac{1}{2\pi i}\int\limits_{L_2}\frac{N'(t)}{t-z}\,dt. \tag{11a}$$

The function $\eta(z)$ is regular outside L_2 and equals zero in the infinity. Let us denote by $\eta_i(t)$ and $\eta_e(t)$ the limit values of the function $\eta(z)$, when the point z tends from within and from outside, respectively, to an arbitrary point t of the arc L_2. We get

$$\eta_i(t)=\frac{N'(t)}{2}+\frac{1}{2\pi i}\int\limits_{L_2}\frac{N'(t_1)}{t_1-t}\,dt_1, \tag{11b}$$

$$\eta_e(t)=-\frac{N'(t)}{2}+\frac{1}{2\pi i}\int\limits_{L_2}\frac{N'(t_1)}{t_1-t}\,dt_1. \tag{11c}$$

Summing up the equations (11b) and (11c), we find that

$$\eta_i(t)+\eta_e(t)=\frac{Pi}{2\pi}\left(\frac{1}{t^2}-\frac{1}{R^2}\right)+\frac{b_1}{t}. \tag{11d}$$

The function $\eta(z)$ will be sought in the form

$$\eta(z)=\frac{F(z)}{\sqrt{(z-a_1)(z-a_2)}},$$

where a_1 and a_2 are the ends of the contact arc L_2 ($a_2=-\bar{a}_1$). We then put

$$\eta_i(t)=\frac{F(t)}{\omega(t)};\quad \eta_e(t)=\frac{F(t)}{-\omega(t)}.$$

Here the notation $\omega(t)=\sqrt{(t-a_1)(t-a_2)}$ is introduced.
The equation (11d) may now be rewritten as follows:

$$F_i(t)-F_e(t)=\omega(t)\left[\frac{Pi}{2\pi}\left(\frac{1}{t^2}-\frac{1}{R^2}\right)+\frac{b_1}{t}\right]. \tag{11e}$$

Solving the equation (11e), we find

$$F(z)=\omega(z)\left[\frac{Pi}{4\pi}\left(\frac{1}{z^2}-\frac{1}{R^2}\right)+\frac{b_1}{2z}\right]+$$

$$+\frac{Pi}{4\pi}\left(-\frac{iR}{z^2}+\frac{\frac{a_1-a_1}{2iR}}{z}+\frac{z-\frac{a_1-a_1}{2}}{R^2}\right)-b_1\frac{z+iR}{2z}. \qquad (11)$$

From (11b) and (11e) we have

$$N'(t)=\eta_i(t)-\eta_e(t)$$

$$N'(t)=\frac{1}{\omega(t)}\left[\frac{Pi}{2\pi}\left(-\frac{iR}{t^2}+\frac{\frac{a_1-a_1}{2iR}}{t}+\frac{t-\frac{a_1-\bar{a}_1}{2}}{R^2}\right)-b_1\frac{t+iR}{2t}\right]. \qquad (12)$$

Hence

$$N(t)=\frac{Pi}{2\pi R^2}[\omega(t)-\bar{\omega}(t)]-b_1\lg\frac{t-iR+\omega(t)}{t-iR-\omega(t)}. \qquad (13)$$

The constant b_1 and the ends of the contact arc L_2 are determined from the following equations:

$$b_1=\frac{1}{4\pi i}\int_{L_2}\frac{N(t)}{t}\,dt, \qquad (14)$$

$$P=-\int_{L_2}N(t)\,dt, \qquad (15)$$

The joint solution of these equations gives:

$$b_1=-0.32\frac{P}{\pi R}, \qquad (16)$$

$$\frac{a_1-\bar{a}_1}{2iR}=-0.09. \qquad (17)$$

This completes the solution of the problem.

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MATHEMATICAL PHYSICS

ON OPERATIONAL SOLUTIONS OF SYSTEMS OF LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

By B. V. BULGAKOV

(Communicated by A. N. Kolmogoroff, Member of the Academy, 9. IV. 1943)

Operational Calculus combined with matrix notations provides a concise apparatus for the construction of effective solutions of systems of linear differential equations with constant coefficients satisfying the given initial conditions.

Formulae of this type can by found in the book by Frazer, Duncan and Collar (1), being derived for the problems referred to by the authors as the standard type. Panovko (2) has especially considered mechanical conservative systems. For the most general forms of equations, however, such as are encountered, for instance, in problems of automatic control, no similar work appears to have been accomplished.

Let us denote by D the «indefinite» differential operator d/dt and by $f(D)$, $g(D)$ the two matrices, whose elements $f_{jk}(D)$, $g_{j\varrho}(D)$ are given polynomials in D of any finite degrees with constant coefficients, $f(D)$ is a square matrix of order n, but $g(D)$ is in general a rectangular matrix of order n by r.

Denoting further by $x_1, \ldots, x_n$ the n unknowns and by $y_1(t), \ldots, y_r(t)$ the given perturbatory functions, we introduce two column matrices

$$x = \begin{Vmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{Vmatrix}, \quad y(t) = \begin{Vmatrix} y_1(t) \\ \cdot \\ \cdot \\ y_r(t) \end{Vmatrix}. \tag{1}$$

The system to be considered is

$$\sum_{k=1}^{n} f_{jk}(D)\, x_k = \sum_{\varrho=1}^{r} g_{j\varrho}(D)\, y_\varrho(t), \quad j = 1, \ldots, n \tag{2}$$

or, expressed in matrices,

$$f(D)\, x = g(D)\, y(t). \tag{3}$$

The matrix $f(D)$ is supposed to be non-singular so that its determinant $\Delta(D) = \det f(D)$ does not vanish identically, but no further assumptions relative to $f(D)$ are made. Let $D^{m_j} x_j$ be the highest of all those derivatives of a

certain unknown x_j, as occur in the equations, and denote by L the greatest of the integers $m_1, \ldots, m_n$. Then $f(D)$ can be expressed as a matrix polynomial

$$f(D) = a_0 D^L + a_1 D^{L-1} + \ldots + a_{L-1} D + a_L . \tag{4}$$

If, in particular, $g(D) = E$, where E is the unit matrix of order n, then the right-hand sides of (2) are simply $y_j(t)$.

Following Carson and van der Pol ([3–5]) and denoting by p the «definite» operator, we will regard $X(p)$ as the operational representation of any scalar or matrix function $x(t)$, if $X(p)$ and $x(t)$ are related by the equation of the Laplacian transformation

$$X(p) = p \int_0^\infty e^{-pt} x(t)\, dt$$

under usual assumptions. In Grammel's notation ([6]) this relation is expressed symbolically as

$$X(p) \doteq x(t).$$

By means of the known formula ([7])

$$D^m x \doteq p^m \left[X(p) - x(0) - \frac{1}{p} x(0) - \ldots - \frac{1}{p^{m-1}} \overset{m-1}{x}(0) \right] \tag{5}$$

we have then

$$f(D)\, x \doteq f(p)\, X(p) - pP(p) \tag{6}$$

with the column matrix

$$P(p) = \frac{1}{p} f_0^-(p)\, x(0) + \frac{1}{p^2} f_1^-(p)\, x(0) + \ldots + \frac{1}{p^L} f_{L-1}^-(p) \overset{L-1}{x}(0), \tag{7}$$

the polynomials $f_0^-(p)$, $f_1^-(p)$, ... being derived from $f(p)$ by cancelling one, two, ... of its last terms. Therefore putting $D_0 \equiv d/dt_0$ and denoting by h any non-negative integer, we can express $P(p)$ in a different form, more convenient for further use

$$P(p) = \frac{1}{p} \left\{ f(p) - a_L + [f(p) - (a_{L-1} p + a_L)] \frac{D_0^0}{p^0} + \ldots + [f(p) - \right.$$

$$- (a_1 p^{L-1} + \ldots + a_{L-1} p + a_L)] \frac{D_0^{L-1}}{p^{L-1}} + [f(p) - (a_0 p^L + a_1 p^{L-1} + \ldots + a_{L-1} p +$$

$$\left. + a_L)] \left(\frac{D_0^L}{p^L} + \ldots + \frac{D_0^{L+h}}{p^{L+h}} \right) \right\} x(0) = \frac{1}{p} \left[f(p) \left(1 + \frac{D_0}{p} + \ldots + \frac{D_0^{L+h}}{p^{L+h}} \right) - \right.$$

$$- a_0 D_0^L \left(1 + \frac{D_0}{p} + \ldots + \frac{D_0^h}{p^h} \right) - a_1 D_0^{L-1} \left(1 + \frac{D_0}{p} + \ldots + \frac{D_0^{h+1}}{p^{h+1}} \right) - \ldots -$$

$$\left. - a_L \left(1 + \frac{D_0}{p} + \ldots + \frac{D_0^{L+h}}{p^{L+h}} \right) \right] x(0);$$

or

$$P(p) = \frac{1}{p} \left\{ f(p) \left(1 + \frac{D_0}{p} + \ldots + \frac{D_0^{L+h}}{p^{L+h}} \right) - f(D_0) \left(1 + \frac{D_0}{p} + \ldots + \frac{D_0^h}{p^h} \right) - \right.$$

$$\left. - \frac{D_0^{h+1}}{p^{h+1}} \left[f_{L-1}(D_0) + f_{L-2}(D_0) \frac{D_0}{p} + \ldots + f_0(D_0) \frac{D_0^{L-1}}{p^{L-1}} \right] \right\} x(0) \tag{8}$$

with

$$f_0(p)=a_L,\ f_1(p)=a_{L-1}p+a_L,\ \ldots,\ f_L(p)=f(p). \tag{9}$$

Similarly

$$g(D)\,y(t) \doteq g(p)\,Y(p)-pQ(p) \tag{10}$$

with the column matrix

$$Q(p)=\frac{1}{p}\left\{ g(p)\left(1+\frac{D_0}{p}+\ldots+\frac{D_0^{M+h}}{p^{M+h}}\right)-g(D_0)\left(1+\frac{D_0}{p}+\ldots+\frac{D_0^{h}}{p^{h}}\right)- \right.$$
$$\left. -\frac{D_0^{h+1}}{p^{h+1}}\left[g_{M-1}(D_0)+g_{M-2}(D_0)\frac{D_0}{p}+\ldots+g_0(D_0)\frac{D_0^{M-1}}{p^{M-1}}\right]\right\} y(0) \tag{11}$$

where M is the highest degree of D in $g(D)$.

The operational equivalent of the equation (3) is thus

$$f(p)\,X(p)=p\,[P(p)-Q(p)]+g(p)\,Y(p); \tag{12}$$

and hence, solving formally for $X(p)$ and denoting by $F(D)$ the adjoint matrix of $f(D)$,

$$X(p)=p\frac{F(p)\,[P(p)-Q(p)]}{\Delta(p)}+\frac{F(p)\,g(p)}{\Delta(p)}\,Y(p). \tag{13}$$

Now let λ_ν, $\nu=1,\ \ldots,\ N$ be the distinct roots of $\Delta(p)$, and q_ν their respective degrees of multiplicity. Developing in partial fractions, we obtain

$$\frac{F(p)\,[P(p)-Q(p)]}{\Delta(p)}=-R(p)+A(p),\qquad \frac{F(p)\,g(p)}{\Delta(p)}=S(p)+B(p), \tag{14}$$

where $-R(p)$ and $S(p)$ denote the respective integral quotients, whereas $A(p)$ and $B(p)$ are the fractional sums. The first of these sums is

$$A(p)=\sum_{\nu=1}^{N}\sum_{\varkappa=0}^{q_\nu-1}\frac{A_{\varkappa+1}^{(\nu)}}{(p-\lambda_\nu)^{\varkappa+1}} \tag{15}$$

with the column matrices

$$A_\varkappa^{(\nu)}=\frac{1}{(q_\nu-\varkappa)!}\left\{\frac{d^{q_\nu-\varkappa}}{dp^{q_\nu-\varkappa}}\,\frac{F(p)\,[P(p)-Q(p)]\,(p-\lambda_\nu)^{q_\nu}}{\Delta(p)}\right\}_{p=\lambda_\nu},\qquad \varkappa=1,\ \ldots,\ q_\nu.$$

The operational formula ([8])

$$p\,\frac{d^\varkappa}{dp^\varkappa}\,\frac{\Phi(p)}{p}\doteq(-)^\varkappa t^\varkappa\varphi(t),$$

when applied to the function $\varphi(t)=\exp\lambda_\nu t$ and its operational representation $\Phi(p)=p/(p-\lambda_\nu)$, yields

$$\frac{p}{(p-\lambda_\nu)^{\varkappa+1}}\doteq\frac{1}{\varkappa!}\,t^\varkappa\exp\lambda_\nu t \tag{16}$$

and

$$p\sum_{\varkappa=0}^{q_\nu-1}\frac{A_{\varkappa+1}^{(\nu)}}{(p-\lambda_\nu)^{\varkappa+1}}\doteq\sum_{\varkappa=0}^{q_\nu-1}\frac{t^\varkappa\exp\lambda_\nu t}{\varkappa!\,(q_\nu-1-\varkappa)!}\left\{\frac{d^{q_\nu-1-\varkappa}}{dp^{q_\nu-1-\varkappa}}\,\frac{F(p)\,[P(p)-Q(p)]\,(p-\lambda_\nu)^{q_\nu}}{\Delta(p)}\right\}_{p=\lambda_\nu}=$$
$$=\frac{\exp\lambda_\nu t}{(q_\nu-1)!}\left\{\left(t+\frac{d}{dp}\right)^{q_\nu-1}\frac{F(p)\,[P(p)-Q(p)]\,(p-\lambda_\nu)^{q_\nu}}{\Delta(p)}\right\}_{p=\lambda_\nu}.$$

Therefore

$$pA(p) \doteq \alpha(t) \tag{17}$$

with the column matrix

$$\alpha(t) = \sum_{\nu=1}^{N} \frac{\exp \lambda_\nu t}{(q_\nu - 1)!} \left\{ \left(t + \frac{d}{dp} \right)^{q_\nu - 1} \frac{F(p)\,[P(p) - Q(p)]\,(p-\lambda_\nu)^{q_\nu}}{\Delta(p)} \right\}_{p=\lambda_\nu}. \tag{18}$$

Similarly

$$pB(p) \doteq \beta(t), \tag{19}$$

$$\beta(t) = \sum_{\nu=1}^{N} \frac{\exp \lambda_\nu t}{(q_\nu - 1)!} \left\{ \left(t + \frac{d}{dp} \right)^{q_\nu - 1} \frac{F(p)\,g(p)\,(p-\lambda_\nu)^{q_\nu}}{\Delta(p)} \right\}_{p=\lambda_\nu}, \tag{20}$$

where $\beta(t)$, as well as $B(p)$, is a rectangular matrix of order n by r. In virtue of the multiplication theorem of the Operational Calculus (*) we obtain

$$B(p)\,Y(p) = \frac{1}{p}\,pB(p)\,Y(p) \doteq \int_0^t \beta(t-\tau)\,y(\tau)\,d\tau. \tag{21}$$

The operational solution (13) may be written as

$$X(p) = pA(p) + S(p)\,Y(p) - pR(p) + B(p)\,Y(p). \tag{22}$$

To acquire the interpretation of the second and third terms on the right we use (8), (11), the identity $F(p)\,f(p) = \Delta(p)\,E$, the second formula (14) and the equation $f(D_0)\,x(0) = g(D_0)\,y(0)$, satisfied by the initial values. Then

$$\frac{F(p)\,[P(p)-Q(p)]}{\Delta(p)} = \left(\frac{1}{p} + \frac{D_0}{p^2} + \ldots + \frac{D_0^{L+h}}{p^{L+h+1}} \right) x(0) -$$

$$- [S(p) + B(p)] \left(\frac{1}{p} + \frac{D_0}{p} + \ldots + \frac{D_0^{M+h}}{p^{M+h+1}} \right) y(0) +$$

$$+ \frac{F(p)\,D_0^{h+1}}{\Delta(p)\,p^{h+2}} \left\{ - \left[f_{L-1}(D_0) + f_{L-2}(D_0)\,\frac{D_0}{p} + \ldots + f_0(D_0)\,\frac{D_0^{L-1}}{p^{L-1}} \right] x(0) + \right.$$

$$\left. + \left[g_{M-1}(D_0) + g_{M-2}(D_0)\,\frac{D}{p} + \ldots + g_0(D_0)\,\frac{D_0^{M-1}}{p^{M-1}} \right] y(0) \right\}.$$

The integer h can here be taken as large as to render the degree of $\Delta(p)\,p^{h+2}$ in p higher than that of $F(p)$, and then it appears that the integral constituent of the rational function last written, if any, is originated exclusively by the terms

$$-S(p) \left(\frac{1}{p} + \frac{D_0}{p^2} + \ldots + \frac{D_0^{M+h}}{p^{M+h+1}} \right) y(0).$$

This integral constituent, with its sign reversed, is evidently

$$R(p) = \frac{1}{p} S_0^-(p)\,y(0) + \frac{1}{p^2} S_1^-(p)\,\dot{y}(0) + \ldots \tag{23}$$

where the polynomials $S_0^-(p)$, $S_1^-(p)$, ... are formed in the same way as the functions $f_0^-(p)$, $f_1^-(p)$, ... above. Comparing with (7) it is seen that $R(p)$ is connected with $S(D)$ and the initial values of y and its derivatives in the same manner as $P(p)$ was connected with $f(D)$ and the initial values of x and its derivatives.

It follows that

$$S(p)\,Y(p) - pR(p) \dot{\to} S(D)\,y(t). \tag{24}$$

With the aid of (17), (21) and (24) the operational relation (22) is finally interpreted as

$$x(t) = \alpha(t) + S(D)\,y(t) + \int_0^t \beta(t-\tau)\,y(\tau)\,d\tau \tag{25}$$

which is the solution of the problem. By means of $\alpha(t)$ it involves, for each j, the initial values of x_j and its derivatives up to and including that of the order $m_j - 1$, and these values are actually assumed for $t = 0$. But they cannot all be assigned independently, if the determinant of the coefficients of the highest derivatives $D^{m_j} x_j$ happens to vanish.

For further details and examples reference is made to a supplementary publication in the «Applied Mathematics and Mechanics» (Russian).

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TECHNICAL PHYSICS

DEGREE OF POLARIZATION OF LIGHT ON LATERAL RADIATION

By A. GERSHUN

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The loss of energy by reflection on passing through the surface of the radiating body depends upon the direction of the electric vector. Therefore the light emitted to the outside is polarized even when each elementary volume gives out non-polarized light.

If the radiating surface is smooth (polished, for instance), the brightness and the degree of polarization of the radiated light change appreciably with the angle of emission α, formed by the ray with the normal. In case $\alpha \to \frac{\pi}{2}$, the brightness $B(\alpha) \to 0$, and the degree of polarization $P(\alpha)$ tends to the limiting value

$$P\left(\frac{\pi}{2}\right) = \lim_{\alpha \to \frac{\pi}{2}} \frac{B_p(\alpha) - B_s(\alpha)}{B_p(\alpha) + B_s(\alpha)}, \qquad (1)$$

where $B_p(\alpha)$ and $B_s(\alpha)$ are the brightness components relative to the states of polarization (index p defines the position of the electric vector as lying in the plane containing the ray and the normal, index s indicates the position perpendicular to this plane).

If the product of the thickness of the radiating layer by the absorption coefficient of the proper radiation is large, then, supposing Kirchhoff's law to be valid, we shall have

$$\frac{B_p(\alpha)}{B_s(\alpha)} = \frac{1 - r_p(\alpha)}{1 - r_s(\alpha)}, \qquad (2)$$

where $r_p(\alpha)$ and $r_s(\alpha)$ are the respective values of the coefficient of reflection.

If now the formulae used in the optics of metals [1] to express the coefficient of reflection by the refraction index n and absorption constant $\varkappa$ (the complex index of refraction $n - i\varkappa$) are applied, it is possible, starting from the relations written above, to show that

$$P\left(\frac{\pi}{2}\right) = \frac{\sqrt{(n^2 - n^2\varkappa^2 - 1)^2 + 4n^4\varkappa^2}}{2 + \sqrt{(n^2 - n^2\varkappa^2 - 1)^2 + 4n^4\varkappa^2}}. \qquad (3)$$

Taking, to put an example, for incandescent turgsten $n = 3.85$ and $\varkappa = 0.89$,

we get $P\left(\frac{\pi}{2}\right) = 0.93$, which agrees with the measurements and computations made by Worthing (2).

In the absence of metallic absorption ($\varkappa \ll 1$) we have, as a particular case of formula (3):

$$P\left(\frac{\pi}{2}\right) = \frac{n^2-1}{n^2+4} \quad (4)$$

Formula (3) can as well be derived directly from the relations (1) and (2) and the approximate expressions of Fresnel's formulae as formerly obtained by the author for the case of reflection at a large angle (3). When allowance is made for repeated refractions, formula (4) holds for a plane-parallel radiating dielectric layer of any thickness great in comparison with the wave-lengths involved.

State Optical Institute.

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CHEMISTRY

A NEW WATER SOLUBLE ANALOGUE OF VITAMIN K

By A. V. PALLADIN, Member of the Academy

Antihaemorrhagic activity is shown not only by the natural vitamin K_1 (2-methyl-3-phytyl-1, 4-naphthaquinone) found in green parts of plants, and by vitamin K_2 (2-methyl-3-hexamethyl-tetracosahexaenyl-1,4-naphthaquinone) produced by bacteria, but by naphthaquinone derivatives of a simpler structure, as well, such as, for instance, 2-methyl-1, 4-naphthaquinone to which the author has proposed to give the name of vitamin K_3[1]. The latter is not only the most active among all the analogues of vitamin K_1[2], but is twice as active antihaemorrhagically as vitamin K_1 itself.

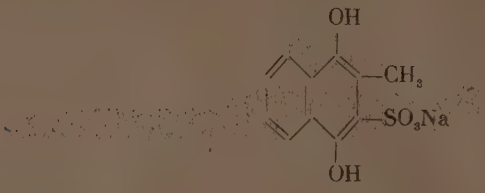
As vitamin K_1 is insoluble and methyl-naphthaquinone but very poorly soluble in water, this making it difficult to use them parenterally, lately many attempts have been made to prepare water soluble analogues of vitamin K and, in particular, water soluble derivatives of 2-methyl-1, 4-naphthaquinone. Ordinarily esters of different acids and of this compound were obtained. Thus have been synthetized sodium 2-methyl-1,4-naphthahydroquinone disulphate [3], sodium 2-methyl-1,4-naphthahydroquinone diphosphate [4] etc. [5]. These esters (their sodium salts) were soluble in water, but their physiological activity was considerably below that of methyl-naphthaquinone, and, besides, the data of different authors concerning the activity of the same preparations were dissimilar. In the USSR such a preparation, potassium 2-methyl-1, 4-naphthaquinone-sulphonate has been synthetized in the All-Union Institute of Experimental Medicine (VIEM). This preparation, according to data by Kudriashov [6], was one and a half — two times less active than methyl-naphthaquinone.

The problem of obtaining a water soluble analogue of vitamin K became particularly actual when our investigations [7] showed that 2-methyl-1, 4-naphthaquinone (vitamin K_3) had the property of stopping not only haemorrhages connected with hypoprothrombinaemia [as it was considered by all the authors, beginning with Dam and Quick [8]], but also normal parenchymatous haemorrhages with a normal content of prothrombin in blood (the possibility of the latter having been denied until lately), and to speedy up the healing of wounds.

This incited the author to work at the preparation of a water soluble derivative of 2-methyl-1, 4-naphthaquinone, and such a one that could be produced on a large scale in the USSR. As all the esters of methyl-naphthaquinone mentioned above had a lower activity compared to methyl-napthaquinone itself, and at the same time certain materials deficient at present were necessary for their preparation, another course was selected. The possibility has been investigated of synthetizing such a preparation through the action of sodium-bisulphite upon methyl-naphthaquinone, i. e. of obtaining water soluble bisulphite derivatives of methyl-naphthaquinone.

Moore [9] by heating methyl-naphthaquinone with bisulphite solutions obtained a solution possessing antihaemorrhagic activity. According to Moore, such a solu-

tion contained (as an antihaemorrhagic active product of the reaction between methyl-naphthaquinone and bisulphite) sodium 2-methyl-1, 4-naphthahydroquinone-3-sulphonate, having the structure:



Moore was unable to isolate this compound from the solution in a crystalline state. Therefore it was impossible for him to check his opinion concerning the structure of the product in solution with the help of sufficient experimental material.

In the present investigations carried out at the Institute of Biochemistry of the Academy of Sciences of the Ukrainian SSR in Ufa and in the laboratory of the Ufa Vitamin Factory, 2-methyl-1, 4-naphthaquinone (vitamin K_3) produced by the latter * and checked chemically and biologically by the author was used.

First of all, we obtained the product of the reaction of methyl-naphthaquinone with sodium bisulphite in solution, having confirmed thus the possibility of dissolving methyl-naphthaquinone in a sodium bisulphite solution. With this purpose in mind a solution of sodium bisulphite (10 per cent) was prepared in a flask heated on a water bath to 60° and to it 2-methyl-1, 4-naphthaquinone was added (1 mole of methyl-naphthaquinone per 1.5 mole of sodium bisulphite). The flask was shaken; heating on the water bath was continued until the complete dissolution of methyl-naphthaquinone, 3—5 min. being usually needed. A solution coloured light-yellow was obtained **.

The biological tests of this solution showed it to possess an antihaemorrhagic activity equal to the activity of an equivalent alcohol solution of methyl-naphthaquinone and to be at least twice less toxic.

Further, contrary to the indications of Moore concerning his failure to isolate from the solution the product of the reaction of sodium bisulphite with methyl-naphthaquinone in a solid (crystalline) state, the author together with the head of the laboratory of the Vitamin Factory D. Lipovsky and the chemist G. Andreenko were successful in isolating the substance in a crystalline state. It proved only necessary to prepare a more concentrated (not below 20 per cent) solution of the bisulphite derivative of methyl-naphthaquinone and after complete solution of the latter to cool the solution with snow. Crystallization begun with cooling. On the morrow the precipitate was filtered off by suction. After drying (at room temperature) a colourless fine crystalline powder was obtained, insoluble in dry alcohol or acetone, but readily soluble in water.

What was the nature of this crystalline product? One could imagine that the reaction of methyl-naphthaquinone with sodium bisulphite led to the addition of the latter to methyl-naphthaquinone and to the formation of the sodium salt of the bisulphite derivative, soluble in water and having the composition $C_{11}H_8O_2 \cdot NaHSO_3$.

The determination of the sodium content in our crystalline product confirmed the correctness of such a molecular formula. Below, as an example, the results of four analyses are given.

1. Weighed portion 38.24 mg was treated with a drop of conc. sulphuric acid in a crucible and carefully ignited to dark-red incandescence. Obtained Na_2SO_4, 9.81 mg.

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* The production of methyl-naphthaquinone was started by the author in the Ufa Vitamin Factory in the beginning of 1942. The product was manufactured and sold under the name of «vitamin K_3».

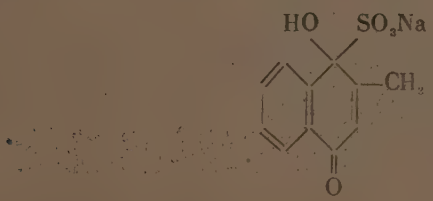
** In this part of the work the author was helped by the laboratory assistant of the Vitamin Factory B. M. Mikhelovich.

II. Weighed portion 27.56 mg; obtained Na_2SO_4, 7.07 mg.
III. Weighed portion 121.40 mg; obtained Na_2SO_4, 31.30 mg.
IV. Weighed portion 68.35 mg; obtained Na_2SO_4, 17.41 mg.
Mol. weight of $C_{11}H_8O_2 \cdot NaHSO_3$, 276. Content of Na: calculated 8.30 per cent; found: I—8.29 per cent; II—8.31 per cent; III—8.35 per cent; IV—8.26 per cent. Average 8.30 per cent.

Is the crystalline product $C_{11}H_9O_5SNa$ obtained by us by its structure a typical bisulphite complex or the structure of a sulphonic acid should be ascribed to it, as it is done by Moore? Two experiments carried out by us make it possible to give a definite answer to this question.

When to a colourless transparent solution of the crystalline product dilute sulphuric acid is added and the solution is heated, the odour of sulphur dioxide appears, the solution becomes yellow, and after cooling a yellow precipitate of methyl-naphthaquinone separates. When to the aqueous solution of the product a sodium carbonate solution is added, there appears immediately a yellow precipitate of the quinone.

The results of these experiments exclude the possibility of ascribing to our crystalline product the structure of methyl-naphthahydroquinone-3-sulphonic acid and are in favour of the bisulphite derivative being a typical bisulphite complex* having the structure:



For this new water soluble analogue of vitamin K, obtained by us in a crystalline state; I propose the name «vikasol».

In order to accelerate the process of the preparation of vikasol (its crystallization), we used later more concentrated solutions, viz. 45 g of sodium bisulphite were dissolved in 100 cm^3 of water, the solution was heated on a water bath to 50—60°; to the warm solution 65 g of methyl-naphthaquinone were added, the solution was shaked and heated on a water bath until methyl-naphthaquinone was completely dissolved (this lasting about 5 min.). Then the flask was surrounded by ice in order to cool the solution to 0°. After 1 hour a considerable crystalline precipitate was filtered off by suction and washed on the Büchner funnel with acetone (200 cm^3). After drying the yield of vikasol was 40 g.

If the solution of methyl-naphthaquinone is carried out at a higher temperature and the solution is heated for a longer time, a solution that is less active antihaemorrhagically is obtained and a smaller amount of vikasol crystallizes from it. Probably, at a higher temperature vikasol is converted into some other isomer of the bisulphite derivative of methyl-naphthaquinone, that does not crystallize when cooled. Perhaps, Moore had obtained such an isomer (he had not described in details the conditions of his experiment) that he failed to crystallize and to which he ascribed the structure of methyl-hydroquinone-sulphonic acid.

Vikasol, being a bisulphite derivative of 2-methyl-1, 4-naphthaquione, having the structure of a bisulphite complex, is a white crystalline powder, well soluble in water, having a bitter taste, no odour and not irritating the mucous in aqueous solution, stable in storage. Thus, the water soluble vikasol has many advantages over the alcohol soluble methyl-naphthaquinone, to say nothing about the possibility of using it for intervenous injections. When used *per os*, neither

* Our thanks are due to prof. A. I. Kiprianov, Corresponding Member of the Academy of Sciences of the Ukrainian SSR, for his help in the chemical investigation of the product obtained.

alcohol, oil, nor cholic acids are necessary, as the aqueous solution of vikasol is absorbed perfectly.

Particular importance is attached to vikasol in the light of our investigations of its antihaemorrhagic properties. We discovered that by using vikasol it becomes possible to stop not only haemorrhages connected with hypoprothrombinaemia but different parenchymatous haemorrhages as well, arising from wounds or after surgical intervention or in connexion with injuries of vessels (when the content of prothrombin in blood is usually normal). Finally, vikasol speeds up the healing of wounds. The possibility of such a wide application of vikasol and of methyl-naphthaquinone (which has been proved previously by us) was until lately denied both for vitamin K_1 and for its analogues.

In order to make the use of vikasol simpler and more accessible we prepared in the Laboratory of the Ufa Vitamin Factory vikasol in tablets with glucose, containing 10 mg of the preparation (calculated for methyl-naphthaquinone) per tablet.

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CHEMISTRY

ISOLATION OF VITAMIN B_1 FROM RICE

By T. E. PAVLOVSKAYA and J. M. SLOBODIN

(Communicated by A. E. Arbusov, Member of the Academy, 19. IV. 1943)

The high requirement for vitamin B_1 makes it necessary not only to prepare aneurin synthetically, but also to use natural products for obtaining concentrates of vitamin B_1 in complete analogy with what we have at present in respect of vitamin C. This problem is also of considerable importance, because such concentrates contain along with aneurin other vitamins of group B, this increasing the therapeutic value of the concentrates.

As raw materials for the preparation of such concentrates yeast is usually recommended and more seldom waste products of the rice industry, although it is just in rice bran that vitamin B_1 was first discovered, and using this material Jansen and Donath (1) achieved in 1926—27 such a considerable success, the result of which was the isolation of almost pure aneurin.

At present the majority of the methods proposed for the extraction of vitamin B_1 from rice waste products are patented, and often it is difficult to understand what is spoken of: the husk of rice, the seed-coat with the embryo, or their mixture.

According to an old German patent dated 1912 alcohol is recommended for the extraction of vitamin B_1 from rice (2). In another patent, issued somewhat later (3), the extraction is carried out with very dilute alcohol (8 per cent). In this case fat and lipoids, as well as many ballast substances are not extracted. In a supplement to this patent (4) it is proposed to subject the initial material to fermentation prior to extraction.

In another group of patents it is recommended to extract vitamin B_1 in acid media. Thus, in one patent (5) the extraction is carried out with 15 per cent alcohol acidified with sulphuric acid to pH = 4.5. In another case 25 per cent alcohol acidified with acetic acid is used for extraction (6).

In counterbalance to these instructions papers exist in which it is recommended to extract vitamin B_1 not with such dilute alcohol solutions, but with more concentrated ones. The most complete extraction of vitamin B_1, according to such data, can be achieved with acidified 70 per cent alcohol.

The purpose of the present investigation was to find out the content of vitamin B_1 in rice husk, bran and embryos, to discover conditions for a most complete extraction of vitamin B_1 and, finally, to work out a method for the production of concentrates. Rice waste products (husk, bran, embryos) were obtained from the Samarkand rice polishing mill. Completeness of extraction was controlled by means of the thiochrome method (7).

The extracts prepared by us had a bitter-sweet taste. Their testing in the clinics of the Military Medical Academy showed the high therapeutic value of these concentrates.

Experimental. The extraction of vitamin B_1 from the material in order to obtain comparable results was carried out as follows.

To 200 g of the material 300 cm^3 of the solvent were added. The mixture was left standing for 3 days. The extract was separated. The residue was carefully washed on a Büchner funnel with the same solvent. From the extract alcohol was distilled off. The aqueous solution, remaining after the distillation of alcohol, was acidified with acetic acid and treated with a 14 per cent solution of lead acetate in order to separate ballast substances. The lead residue was separated and carefully washed with water. The waste liquid was added to the filtrate and the latter was treated with lead carbonate for neutralization. The residue was separated and carefully washed with water. The neutral solution was freed from an excess of lead by passing hydrogen sulphide through it. The precipitated lead sulphate was separated and well washed with water. The filtrate was evaporated to a volume of 50—100 cm^3. After each operation the vitamin remaining in the residues was determined by the thiochrome method. The precipitates were washed until a negative test with thiochrome was obtained.

Table 1 shows the results obtained with extraction of 200 g of rice embryos with 70 and 8 per cent alcohol. It may be seen from this table that 8 per cent alcohol extracts many times as much vitamin as 70 per cent alcohol and does not extract at all fat. 70 per cent alcohol extracted so much fat that when the alcohol was distilled from the extract, two layers were obtained, the upper one—fat—hardening on cooling.

Table 1

Amount of embryos, g	Concentration of alcohol, %	Amount of extracted vitamin B_1, γ	Amount of extracted fat, g
200	70	30	10
200	8	1050	none

Subsequently all the extractions were carried out with dilute alcohol.

It seemed quite natural to expect that finely ground material would give a better yield of the vitamin, all other conditions remaining the same. Table 2 shows results obtained from the extraction of rice embryos used in the form in which they are obtained at the rice mill and as finely ground material. The latter yielded almost 8 times more vitamin.

Table 2

Amount of embryos, g	Time of extraction, days	Amount of extracted vitamin B_1, γ	
		rough grind	fine grind
200	3	1000	—
200	3	—	7600

The dependence of the amount of extracted vitamin upon the time of extraction is shown in Table 3. However, it is difficult to extract all the vitamin contained in the material when using only one portion of the solvent.

Table 3

Amount of germs, g	Time of extraction, days	Amount of extracted vitamin B_1, γ
200	3	7600
200	6	10800

In order to achieve complete extraction it is necessary to repeat many times the extraction using each time a new portion of the solvent. A complete extraction was achieved only after 5 extractions, each lasting 3 days (Table 4).

Table 4

Amount of embryos, g	Extraction of vitamin B_1, γ						Extracted total
	1 extr.	2 extr.	3 extr.	4 extr.	5 extr.	6 extr.	
200	7600	3368	2560	360	160	0	14048

At the same time the comparative study of rice waste products showed that embryos are the richest in vitamin B_1, being much richer than bran and husk (Table 5).

Table 5

Material	Amount, g	Content of vitamin B_1, γ
Embryos	1000	70240
Husk	1000	500
Bran	1000	430

It should also be noted that at present, owing to the separation of embryos from bran in the production process, the value of bran as a vitamin containing material has decreased considerably. At the same time it should be mentioned that the quality of the concentrate obtained from bran and husk is much lower than that of the concentrate obtained from embryos.

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BIOCHEMISTRY

REARRANGEMENT OF CELL PROTOPLAST INDUCED BY COCARBOXYLASE AND THE COMPONENTS OF VITAMIN B_1 (THIAZOLE AND PYRIMIDINE)

By M. N. MEISSEL

(Communicated by L. A. Orbeli, Member of the Academy, 10. VI. 1943)

In 1938 I showed (1) on yeast and yeast-like organisms that the transition from aerobic conditions of existence to relatively anaerobic ones is attended by an essential morphological and physiological rearrangement of the cell's protoplast. According to the conditions under which they were developing, micro-organisms from the same strain were found to assume a type either respiratory or fermentative, both by their morphological and physiological characters.

The respiratory type was distinguished cytologically by a nucleus though small, but rich in thymonucleic acid, by a considerable development of the vacuolar system, a small content of volutin, an extremely profuse chondriome scattered all over the cell. Physiologically these cultures are endowed with a relatively low fermentative capacity.

The fermentative type is characterized by large cell nucleus poor in thymonucleic acid; by a profuse, dense cytoplasm, a considerable content of volutin and glycogen, and a sharply altered, hypertrophied chondriomic apparatus. Physiologically such cultures display an increased fermentative activity.

It was discovered, moreover (2), that for a number of organisms which require for their growth and reproduction vitamin B_1, this vitamin is the main agent responsible for the morphological and physiological rearrangement of cells, and for the formation of cells of the fermentative type. Being added to a synthetic nutrient medium, it promotes the appearance of organisms of the fermentative type, even under aerobic conditions.

E. N. Odinzova (3) has succeeded in showing at this Laboratory that under anaerobic (fermentative) conditions the synthesis and the accumulation of vitamin B_1 by yeast cells goes on more intensely.

As a certain portion of the vitamin B_1 is transformed in the cell into pyrophosphoric ester, the cocarboxylase, it seemed of interest to find out in how far the fermentative rearrangement of the cell observed by us may be brought about also by the cocarboxylase.

On the fungus *Endomyces Magnusii* which had also been used as material for our preceding work, it was found possible to establish that the effect produced by pyrophosphoric ester is quite identical with that of vitamin B_1. The addition of cocarboxylase to Reader's mineral nutrient solution resulted in the appearance of typical cells of a fermentative type, similar to those as had appeared in the presence of vitamin B_1.

Schopfer showed in a series of papers that micro-organisms may be classed into groups, according to whether they require from the external medium a whole molecule of vitamin B_1, or only some of its components. The most widespread are such organisms as require for their normal development the supply from the external medium of a part of a molecule of vitamin B_1, the thiazole (more precisely, 4-methyl-5-β-oxyethyl-thiazole), being able to synthesize the missing part of the molecule of vitamin B_1, pyrimidine. In order to test the influence of the separate components of vitamin B_1 upon the cell of *Endomyces Magnusii*, Reader's nutrient mineral solution was added equimolecular quantities of thiazole, pyrimidine, of a mixture of these components, and crystalline vitamin B_1, separately. After a 48 hours development on such media the cultures were examined cytologically and physiologically. The vitamin B_1, the mixture of thiazole and pyrimidine, and the thiazole were found to be equally able of a fermentative rearrangement of the cell, whereas pyrimidine when added alone proved to produce no effect whatsoever. In the table below are given data as to the fermentative characteristic of cultures of *Endomyces Magnusii* which developed on mineral media with addition of vitamin B_1 and of its components, respectively.

Intensity of Fermentation of *Endomyces Magnusii* as Dependent on the Presence in the Medium of Vitamin B_1 and its Components

Conditions of cultivation	CO_2 evolved in 1 hour per 1 mg of dry substance in mm^3
Medium Reader-agar	8
» » » + pyrimidine	6
Medium Reader-agar + thiazole	34
» » » + (thiazole + pyrimidine)	41
Medium Reader-agar + vitamin B_1	38

Thus, similar to an integer molecule of vitamin B_1, its part, the thiazole, is also able to bring about the fermentative rearrangement of cells in *Endomyces Magnusii*. But is this fermentative cell rearrangement accompanied by the synthesis of vitamin B_1 from the thiazole? We have succeeded in showing by way of direct experiments involving cultivation on a nutrient medium with thiazole in excess that under these conditions synthesis and excessive accumulation of vitamin B_1 in the cells actually take place. Whereas on Reader's nutrient solution without thiazole the culture of *Endomyces Magnusii* was found to contain but traces of vitamin B_1, when cultivated with thiazole the cells of this organism synthesized and accumulated up to 120 micrograms of vitamin per 1 g of dry substance.

It thus should be considered as safely established that vegetable organisms that require for their development no more than a part of the vitamin B_1 molecule, are capable of synthesizing a whole molecule of this vitamin in quantities considerably exceeding the usual requirement of the organism.

An excessive supply of thiazole from without is attended by a correspondingly increased synthesis of the second part of the molecule of vitamin B_1, and by their combining into a whole molecule of the vitamin.

The vitamin B_1 formed in this way may accumulate in the cells in considerable quantities, causing their morphological and physiological rearrangement.

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BIOCHEMISTRY

SYNTHESIS OF VITAMIN B_1 BY CULTIVATED YEASTS FROM THIAZOLE

By E. N. ODINZOVA

(Communicated by L. A. Orbeli, Member of the Academy, 10. VI. 1943)

The investigation of biosynthesis of vitamin B_1 by a vegetable and, in particular, by a microbic cell from simpler compounds is still at its initial stage. All that has until recently been done in this field bears largely on the problem of growth substances (growth-effect), being no direct investigation of the problem referred to above.

Vitamin B_1 is known as the most widespread activizer of reproduction. Its addition in minute amounts to nutrient media was found necessary to stimulate the development of many bacteria, fungi and yeast-like organisms. There are number of microbes, however, that thrive on purely mineral media, too. It is a large-scale revision of various forms of bacteria and fungi made at this angle that gave the first indications as to the possibility of vitamin B_1 being synthesized by micro-organisms. No direct evidence of the synthesis of vitamin B_1 was secured until part of the organisms had been found able to develop normally not only in the presence of vitamin B_1, but of its components, thiazole or pyrimidine, as well.

Particular interest for the question we are dealing with belongs to the so-called cultivated yeasts which are endowed with a high fermentative activity. To these belong species of *Saccharomyces*. A typical feature of this group of micro-organisms is the high content of vitamin B_1 in their cells. This is a character which sharply distinguishes the cultured yeasts from the rest of microbes, as well as from cells of vegetable and animal tissues, even those especially rich in vitamin B_1. However, our investigations [1] have shown that the high content of vitamin B_1 is connected with definite conditions of life-activity: vitamin B_1 is being accumulated in large amount during fermentation, and its content decreases sharply as soon as the culture begins to be freely supplied with oxygen from the air. It is to cultivation conditions in the first line that may be referred the high content of vitamin B_1 in the brewer's yeast (*Saccharomyces Carlsbergensis*), and their relatively small content in the baker's yeast (*S. cerevisiae*).

Our experiments on the conditions and dynamics of the accumulation of vitamin B_1 in the cell made us conclude that the accumulation of vitamin B_1 during fermentation is an active process which is due to the high requirement of this substance experienced by the yeast under conditions of oxygen deficiency.

Information as to the biosynthesis of vitamin B_1 by cultivated yeast is very scanty and contradictory; even in those cases when the synthesis of vitamin B_1 seemed to be safely established [2,3], the vitamin was found in very small quantities. Thus, after having cultivated yeast on a mineral medium for a long period of time, Lassen found but a third of what has usually been observed when cultivating it on a full value medium (beer-wort). It thus remains obscure, whether the high

accumulation of vitamin B_1 (up to 200 micrograms per 1 g of dry substance) is due to an increased synthesizing capacity of the given organisms, or whether the vitamin is being accumulated from the external medium.

Unfortunately, we were not able to cultivate yeast of this type on a full value synthetic medium in the absence of vitamin B_1 *. Use was made, therefore, of the method of a short-lived fermentation in a micro-fermentative apparatus. Owing to the large number of yeast cells (100—200 million per 1 cm^3 of the fermenting liquid), the fermentation in a 4 per cent sugar solution proceeds very intensely, being in the main completed within 2—3 hours. Cell proliferation is almost entirely excluded in this case. We have been prompted to carry on experiments along this line by interesting data obtained with crystalline vitamin B_1. While working with readily fermenting forms (various species of *Saccharomyces*), at the period of preliminary proliferation we never succeeded by biological methods to make them so poor in vitamin B_1, as to accelerate the fermentation process by adding the vitamin from without. In spite of the fact, however, that the yeast investigated by us showed absolutely no external reaction for vitamin B_1, it proved very active in accumulating the crystalline vitamin B_1 added. The content of the latter in the cells can thus be made to amount to 1000 micrograms per 1 g of the dry yeast substance, while the largest quantity of B_1, that has ever been observed by us under conditions of natural fermentation in brewer's yeast, made up no more than about 200 micrograms per 1 g of dry substance.

These experiments with crystalline vitamin B_1 suggested us the idea of studying the synthesis of vitamin B_1 from its components. The first experiments were made with the thiazole** that had kindly been put at our disposal by Prof. O. Magnidson. The yeast was preliminarily left to proliferate on wort-agar during 48 hours; the content of vitamin B_1 in its cells was 20—30 micrograms per 1 g of dry substance. The addition of thiazole was found to have no promoting effect upon the fermentation. After a two-hour fermentation, however, we invariably observed the content of vitamin B_1 in the cells to augment considerably, attaining 200 micrograms per 1 g of the dry yeast substance. Such a large quantity could be formed in no other way than at expense of a synthesis from the thiazole. Its formation must have proceeded in amounts almost equimolecular to those of the thiazole added.

No cases of synthesis of vitamin B_1 from thiazole have up to now been described in literature. A thing that remains obscure is the source of pyrimidine formation. We suppose that this component might have been obtained by the cell by way of some internal displacements.

Laying special stress upon the problem of the synthesis of vitamins by microorganisms, we thought it necessary to publish our first data relating to the synthesis of vitamin B_1 by cultivated yeast. The principle underlying our method is so simple that, certain quantities of thiazole being available, it can and must be widely applied to the practice of mass production from baker's yeast of alimentary yeast rich in vitamin B_1.

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* For its normal development cultivated yeast requires a number of supplementary organic activizers.

** 4-methyl-5-β-hydroxiethyl-thiazole.

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GEOLOGY

ON THE LOWER CRETACEOUS FAUNA OF THE SUCHAN COAL REGION

By V. I. BODYLEVSKY

(Communicated by W. A. Obruchev, Member of the Academy, 15. V. 1943)

In the course of detailed geological surveying carried out in the Suchan coal region by V. N. Vereschaguin and Z. P. Potapova in 1938 and 1939, a rich fauna was discovered in the deposits underlying the Suchan coal-bearing series. This is one of the most happy palaeontological findings in the Mesozoic deposits of the Far East in the recent years; not only it furnishes precise indications as to the lower boundary of the coal-bearing series, but also sheds light upon some general problems of the stratigraphy and palaeogeography of the Mesozoic of this region.

According to V. N. Vereschaguin, the main prospecting line in the upper course of the Alexandrovsky spring, which furnished the most abundant palaeontological material, exposed the following section (in ascending order):

1. Dark-grey, thin-bedded aleurolites and argillites with abundant *Exogyra* and *Corbicula*, 20 m thick.

2. Quartz-feldspar sandstones, rusty brown, compact, with numerous *Aucella*, 45 m thick.

3. Calcareous sandstones, abounding in fragments of *Aucella* and *Exogyra*, 5 m thick.

4. Sandstones of the same type as in layer 2, over 200—300 m thick.

The underlying strata (below horizon 1) have not been directly discovered. The youngest of the subjacent horizons seems to be a marine Upper Jurassic [Vereschaguin's finding of *Aucella* cf. *bronni* (Rouill.) Lah. in the Molchanov region].

Layer 4 is overlain by the coal-bearing series—with an angular unconformity and basal conglomerate at the base.

The author has determined from layer 1: *Exogyra ryosekiensis* Kob. et Suz., *Corbicula* aff. *tetoriensis* Kob. et Suz., *Gastropoda* gen. et sp. indet.

From layer 2: *Aucella bulloides* Lah.; *A.* cf. *terebratuloides* Lah., *A. keyserlingi* Lah., *A.* aff. *keyserlingi* Lah., *A. sibirica* Sok., *A. visingensis* Lah., *A. syzranensis* Pavl., *A. uncitoides* Pavl., *A.* aff. *uncitoides* Pavl.*, *A. wollossowitschi* Sok., *A.* cf. *inflata* (Toula) Lah., *A. sublaevis* Keys., *A. sokolovi* Bodyl., *A.* cf. sp. n. (aff. *bulloides* Lah.), *Perna* aff. *ricordeana* d'Orb. (1 specimen), *Corbicula opiformis* sp. n. **, *C.* aff. *tetoriensis* Kob. et Suz.

From layer 4: *Aucella* sp. indet. (2 specimens), *Exogyra ryosekiensis* Kob. et Suz. (numerous specimens), *Ostrea* cf. *yoshimoensis* Kob. et Suz., *Perna* aff. *ricor-*

* More protuberant and with more regular outlines than in the type.
** With beak prominent as in *Opis* (inne nomen), with the height exceeding the length.

deana d'Orb. (1 specimen) *, *Astarte sakawana* Kob. et Suz., *Corbicula opiformis* sp. n.

From the other localities of fauna occurrence the following are worthy of mention:

1. In the region of the Popov spring (in the sandstone): *Aucella* cf. *keyserlingi* Lah., *A. sublaevis* Keys. (cf. var. *majuscula* Lah.), *A.* sp. n. (aff. *bulloides* Lah.), *Exogyra ryosekiensis* Kob. et Suz., *Ostrea* cf. *yoshimbensis* Kob. et Suz.

2. In the upper course of the Andrianov spring (exposure No. 991, in arenaceous aleurolites): *Exogyra* cf. *ryosekiensis* Kob. et Suz. (numerous, but poorly preserved specimens), *Astarte sakawana* Kob. et Suz., *Aucella* cf. *terebratuloides* Lah. (1 specimen), a natural cast of *Gastropoda*.

In determining the age of the deposits considered, greatest attention belongs to *Aucella* which constitute the bulk of the total fauna and taken as a whole present the best preserved *Aucella* fauna of the Far East. Its age is determined as the Lower Valanginian (the zone of *Tollia stenomphala*) or the lower part of the Middle Valanginian. Synchronous deposits in the Far East are very widespread and are everywhere represented by strata with *Aucella*. Within the boundaries of Sikhote-Alin only up to the present time over ten deposits of the same age have been discovered, but with a fauna considerable poorer and less well-preserved; it is only in the Olguino region that along with specimens of *Aucella* isolated specimens of ammonites—*Polyptychites* (?) sp.—have been also discovered **.

The author's determinations establish the lower age limit for the Suchan coal-bearing formation: it cannot be older than the Middle Valanginian.

Thus, a new indication is obtained for reaching a conclusion as to the age of the flora which characterizes the coal-bearing formation mentioned ***.

The unusual interest presented by the Suchan fauna discussed here lies in the resemblance of some of its elements to the brackish-water fauna recently described by Koboyashi and Suzuki ([3]) from the Ryoseki series of Japan. The common elements are *Exogyra ryosekiensis*, *Ostrea* cf. *yoshimoensis* and *Astarte sakawana*. The brackish-water nature of the Japanese fauna is established from the presence of *Corbicula*, represented also in our fauna. Thus, common nature of the faunas (first discovered by us) furnishes additional grounds for determining the age of Ryoseki—by establishing its contemporaneity with the *Aucella* (Valanginian) beds of the Far East. This is particularly important, because the Lower Cretaceous marine faunas of Japan (with the exception of *Trigonia*) are as yet very little known.

The oldest marine fauna, referring to the Lower Cretaceous, was described from the Sanchu graben ([4]): from the presence of *Simbirskites* its age should be considered to be later (Hauterivian—Barremian) than that of our *Aucella* beds.

It is of interest to note that while *Aucella* are widely distributed in the Lower Cretaceous of the Soviet Far East, they are entirely absent at the same time in the more southerly portions of the West-Pacific geosyncline. On the other hand, in the *Aucella* beds in our country so far no *Exogyra* have been discovered. Considering all the information available on this genus, the assumption may be

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* From the type of d'Orbigny [([1]), p. 494, plate 399] it is distinguished by a greater relative height, which is especially well pronounced at the young stages of growth and is the cause of a more slender shape of the shell.

** Outside the boundaries of the Far East the most closely resembling fauna is the rich *Aucella* fauna described by D. N. Sokolov from the basin of the Anabar river.

*** The upper age limit for the coal-bearing formation may be established, on the one hand, from the occurrence in its upper portions of *Aralia lucifera* (Aptian—Albian, according to A. N. Krishtofovich), and on the other hand, from the presence of fauna in the marine strata overlying it. This fauna was determined by N. S. Voronetz, who came to the conclusion as to its Cenomanian age. It should be noted, however, that the composition of the fauna, as it is easily seen from the list published ([2]) does not justify the adherence to this view: almost all the forms of this list are either new species or have been determined only up to genus, and sometimes with a note of interrogation, as *Tapes* (?) sp. Only one form *Callistina pseudoplana* Yabe et Nagao permits a correlation to be made with the previously known horizons, but it belongs to a genus so poorly characterized by external features that its identification could be hardly made with certainty.

made that its absence in the *Aucella* beds of our country is accounted for by the insufficiently warm waters of the *Aucella* Lower Cretaceous sea. At the only point (Suchan) where these two genera have been found together, it may be observed that a profusion of representations of *Exogyra* is accompanied by only isolated specimens of *Aucella*, and vice versa.

The conclusion suggests itself, that the Suchan region was the southernmost point reached by the boreal (*Aucella*) transgression during the Valanginian. At this point the representatives of the fauna of the boreal sea would come in contact with the fauna of the southern sea (*Exogyra*), penetrating here from the side of Japan.

Finally, a number of problems arise in connexion with the joint occurrence of marine forms with brackish-water *Corbicula* in the beds considered. Unfortunately, there are no precise data available either on the distribution of the latter in individual strata, or on their quantitative proportion in the total fauna. The wish is expressed that in the further study of the beds particular attention be paid to this aspect of the problem.

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BOTANY

GROWTH OF *PINUS SILVESTRIS* L. IN SOLONCHAK SOILS

By I. A. KRUPENIKOV

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According to the generally established opinion, *Pinus silvestris* L. belongs to the number of plants distinguished by high sensitivity to easily soluble salts. Only in Hartig's (1) experiments did that tree prove, owing to deep rooting, insensitive to watering the soil with a 3.47 per cent solution of sodium chloride. Contrary to this, the experimental data obtained by Grigoriev (2) and Stepanov (3) point to a rather poor salt-resistance of *Pinus silvestris*, especially at the time of seed germination. According to Kossovich (4), the pine is remarkably sensitive to the soil's content of NaCl, and the effect of 0.039 per cent or more was found by him to be fatal.

Yakubov (5) reports 0.05 per cent as the nearly limiting content of Cl' and SO_4'' in the soil for the normal development of many common woody plants, and only very few trees bear with more. According to Tanfiliev (6), the effect of easily soluble salts is especially harmful in the case of young trees.

In studying the salt-resistance of some woody and shrubby plants growing under natural conditions in the forests of Kazakhtan, on its southern border (7-9), there were recorded instances of pine thriving on saline soil. Not only old pine trees which may have appeared when the soil had not yet been saline, were observed there, but also a rather profuse undergrowth of varying age. A particularly good undergrowth was stated in Naurzum wood on the shores of the lake Kutan-tal where a solonchak soil contains a more or less high amount of moisture and the water table is high. In other regions of this forest several areas of heath-solonchak soil comparatively dry, at least within certain intervals of the growing season, were found to be overgrown by young pine trees ranging from 8 to 10 years in age, and occasionally by old pine trees. On special investigation this soil exhibited a high content of easily soluble salts, by far exceeding the supposed limit for the growth of common woody species.

To illustrate I give a description of one such typical district. On a solonchak heath overgrown predominantly by such plants as *Elymus salsuginosus*, *Hordeum brevisubulatum*, *Inula salicina*, *Statice Gmelini*, etc. a few isolated pines were noticed ranging from 60 to 80 years in age. On the southern border of that district 8 young pines formed a rather compact group. The data on their age, height and new yearly growth are given in Table 1. From this table it may be inferred that these pines have attained a more or less normal height quite consistent with their age, though there is no uniformity of development between separate trees.

The new growth in height is seen to have been very considerable in certain years, but here again the extreme irregularity of data strikes the eye.

Table 1

Nos. of pines	Age in years	Height in cm	Yearly growth in height in cm			
			1937	1938	1939	1940
1	9	115	14	19	17	5
2	8	82	6	8	10	22
3	9	93	5	4	13	24
4	8	64	5	4	11	15
5	9	176	17	16	28	43
6	9	130	16	14	20	20
7	8	92	9	7	14	22
8	9	122	7	10	15	30
Average		109	10	10	16	22

The soil of the described plot (profile 2501) is a heath-solonchak, or humus-solonchak, loamy in its mechanical composition (Table 3). Calculation to the silicate-humus mass reveals a substantial enrichment of the upper horizons in clayey particles.

The underlying bed is sand. The soil is remarkably high in carbonates and organic matter (Table 4).

Table 2

Stands	Classification according to commercial value	Age class	Age	Density of stands	Store in m^3 per ha	Average yearly growth in m^3 per ha	Average height in m	Average diameter in cm
«Solonchak» pines near lake Kutan-tal	III	IV	70	04	90	1.3	16	32
Average for the stand	III,3	III-IV	60	039	76	0.8	—	—
Average of the IV class of age	III,5	IV	70	04	86	1.2	17.5	28

The relatively heavy mechanical composition of the soil and its high content of organic matter are certainly unfavourable factors for the development of pines upon it, for in North Kazakhstan this tree is mainly associated with light sandy or rocky soil containing small quantities of organic matter (no more than 1—2 per cent).

Water extract analyses (Table 4, sampling June 26, 1940) testify to a high content of salt, especially in the upper horizons where the dense residue attains a ratio of 1.38 per cent and the proportion of Cl' is 0.111 per cent. Considerable quantities of chloride are encountered also at a depth of 30—40 cm, with 0.064 per cent of Cl'. The upper horizons are, moreover, characterized by a high total alkalinity (up to 0.172 per cent), and there were noticed alkali traces from sodium carbonates.

Among the cations Na^+ predominates in nearly all horizons. From these data the soil appears quantitatively to be fairly alkaline. The qualitative relation of the ions in the soil solution is unsatisfactory from the physiological point of view, since there is a predominance of Cl' among the anions and of Na^+ among the cations [10].

Table 3

Mechanical Composition of Soil in Profile 2501, per cent in 100 g of Dry Soil

(Pipette method, treatment with 0.05 *N* HCl)

Depth in cm	Hygroscopic water	Loss by treatm with HCl	Fractions							Total of particles <0.01	Calculated to silicate-humus mass	
			>1.0	1—0.25	0.25—0.05	0.05—0.01	0.01—0.005	0.005—0.001	<0.001		<0.01	<0.001
1—7	7.21	40.77	—	0.13	19.34	16.94	3.77	4.21	14.83	22.81	38.6	25.1
10—20	4.28	65.12	—	0.31	13.76	3.69	3.17	3.31	10.64	17.12	48.9	30.0
30—40	0.52	54.12	—	0.17	37.47	0.63	1.93	0.95	4.73	7.61	16.5	10.0
60—70	0.99	0.64	—	9.73	67.87	4.08	4.97	2.55	10.16	17.68	17.7	10.2
110—120	0.16	0.37	—	14.81	78.34	0.56	1.52	0.32	4.08	5.92	6.0	4.1

On the shores of the salt-lake Kutan-tal where the conditions are more or less similar, many old pines (of about 70 years in age) are growing and have a good appearance with crowns of a brighter green as compared to the colour of the pine in the ordinary dry habitations of Naurzum forest. In area the region of the «solonchak» pines measures 1 ha. The data in Table 2 characterize the pine growth of the district and, for the sake of comparison, average figures are also given for the IV class of age and for the stand as a whole. It may be seen in it that several of the indices are higher for the pines of this district than for the pine stands of other habitations (store, average new growth yearly). The small density of the stands is, generally speaking, typical of Naurzum forest. Besides the old pines, undergrowth from 8 to 10 years of age is encountered in great numbers within the district, and also young seedlings of 1938 and 1939. Most of the seedlings that had appeared in 1938—1939 lived until the end of 1940. To take an experimental plot of 1 m² area as an example, of the 10 pine seedlings of 1938 and one dating from 1939 that had been growing on it on Oct. 9, 1939, 7 and 1, respectively, were still there on Oct. 2, 1940. In 1941 and 1942 the number of young trees arisen from seeds was found to have diminished, but those which had survived continued to develop normally.

Toward the autumn of 1940 the seedlings of 1938 measured about 20 cm in height. This is an exceptionally good growth under the conditions prevailing in Naurzum forest. Within the limits of the district considered 2 soil profiles were put down (samples were taken on September 22, 1939)—profile 56 and profile 57.

The water extract data for these profiles are given in Table 4. The alkalinity of the soil is seen from these data to be much higher in this case than in that of profile 2501, and high quantities of salts are here observed not merely in the upper horizons, but also at a comparatively great depth, as deep, indeed, as the main mass of root of old pines. In profile 56 the proportion of Cl' at a depth of 13 to 17 cm is 0.145 per cent; at a depth of 58 to 69 cm, 0.074 per cent. For SO_4'' the respective values are 0.309 and 0.208 per cent. Both Cl' and SO_4'' in this section are predominantly associated with sodium. In the case of profile 57 which is nearer to the lake and at a lower site, the alkalinity is particularly high at a depth of 100—110 cm; here the ratio of Cl' attains 0.213 per cent, of SO_4'', 0.606 per cent; of Na', 0.419 per cent.

It is interesting to note that the thinnest and the most active roots of the pine are just in this layer or somewhat higher above.

To sum up, in the forests of the extreme south of Kazakhstan the pine has definitely adapted itself to alkaline soils with unfavourable physiological conditions. Apparently, we have to deal here with a new ecological form of pine, and its further investigation may lead to a thorough revision of the existing views on the

Table 4

Data on Content of Humus, CO_2 and on Chemical Analysis of Water Extract from Alkaline Soil under Pine, per cent in 100 g of Dry Soil

Profiles	Depth in cm	Water extract data							Humus according to Knopp	CO_2 according to Knopp
		HCO_3'	Cl'	SO_4''	$Ca^{\cdot\cdot}$	$Mg^{\cdot\cdot}$	$Na^{\cdot} + K^{\cdot}$ from difference	Dry residue		
501	1—7	0.172	0.111	0.044	0.042	0.019	0.074	1.38	31.27	12.41
»	10—20	0.082	0.079	0.039	0.037	0.011	0.037	0.29	14.27	21.03
»	30—40	0.032	0.062	0.016	0.012	0.003	0.041	0.10	1.55	19.79
»	60—70	0.042	0.014	0.039	0.005	0.006	0.027	2.34	1.87	0.53
»	110—120	0.027	0.006	0.021	0.007	0.005	0.007	0.75	—	—
56	13—17	0.029	0.145	0.309	0.014	0.008	0.221	0.81	—	—
»	59—69	0.036	0.074	0.208	0.003	0.003	0.136	0.48	—	—
57	10—19	0.016	0.053	0.191	0.030	0.008	0.083	0.45	—	—
»	45—55	0.034	0.026	0.059	0.004	0.003	0.047	0.18	—	—
»	100—110	0.031	0.213	0.606	0.012	0.006	0.419	1.26	—	—

response of the pine to the alkalination of the soil. When this solonchak ecotype of pine will have been investigated in detail against the background of its ecological environment, it will be possible to discuss the question of the introduction of the Naurzum solonchak pine into cultivation on the alkaline soils of other regions.

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PHYSIOLOGY

MORPHOGENIC ACTIVITY OF VARIOUS PARTS OF THE ANTERIOR LOBE OF THE PITUITARY BODY IN MAN

By A. A. WOITKEWITSCH

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In a number of our previous studies [1-3] it has been shown that the two fundamental processes of development in vertebrates, viz. growth and differentiation, depend on hormones which are being produced by the various cells of the anterior lobe of the pituitary body. The processes of differentiation (ripening of gonades, change and formation anew of integuments, metamorphosis in amphibians) are controlled by the active principle produced by the basophilic cells of the anterior lobe, whereas the growth of animals is controlled by a hormone which is formed in the oxyphilic (eosinophilic) cells.

The dependence of the morphogenic processes on the hormones produced by the anterior lobe of the pituitary body has been shown experimentally, by testing biologically areas of the anterior lobe differing as to their microstructure. Differences in the microstructure of the various regions of the anterior lobe, which are expressed in a prevalence in a given region of cells of a definite category, are as a rule associated with different physiological (often morphogenic) processes.

Some animals display, moreover, in a normal state an irregular distribution of basophilic and oxyphilic elements in the anterior lobe of the pituitary gland.

Such a zoning is clearly manifest in birds, basophilic cells being prevalent in the anterior, and oxyphilic, in the posterior lobe of the organ. No less typical is the zoning observed in the anterior lobe of the pituitary gland of large mammals (cattle): in the central part of the organ there is found a small cone-like region where, along with reserve and, partly, oxyphilic cells, basophilic cells are present in prevalent numbers. In the remaining, larger part of the organ, eosinophilic cells prevail, basophilic and reserve cells being also present. In large animals zoning may easily be observed not only in histological preparations, but also in microscopical sections made across a fresh pituitary gland. This circumstance made it possible to isolate with sufficient accuracy regions with prevailing basophilic or eosinophilic elements, and to carry out various biological tests. The results of these tests showed concordantly that the basophilic cells locate the so-called tropal principles, stimulating the activity of a number of ductless glands, while the eosinophilic cells are the producers of the growth hormone.

When we spoke of a morphological isolation of the fundamental structural elements in the anterior lobe of the pituitary gland, we were ready to assume that various animals may show important variations as to the character of isolation of regions rich in basophilic or oxyphilic cells. Special zones may exist where, in the normal state, cambial reserve cells (the main) are being transformed

largely into basophilic cells, while in the rest of the organ they are transformed into eosinophilic ones.

On account of all these considerations we found it expedient to pass to the next step of our work, whose aim was to examine histo- and physiologically the anterior lobe of the pituitary body in various vertebrates.

Human hypophyses were used as material of this work. We supposed to confine ourselves at first to the study of the thyrotropic function of the anterior lobe of human pituitary gland and to pass subsequently to a study of the gonadotropic and other functions. The technique of the work involved no particular difficulties. In case we were to reveal zoning in the distribution of the structural elements in the anterior lobe of the pituitary gland, we would proceed to biological testing of tissues from the different zones on larvae of amphibians.

In our earlier works ([1-3]) for studying the thyrotropic activity of the pituitary body of the test-objects use was made of larvae of *Rana temporaria* and of the axolotl.

As these species are not encountered in the region of Alma-Ata, we had recourse to larvae of *Rana ridibunda* and *Bufo viridis*. In so far as we have never before studied the reaction of these species for the thyrotropic principle, it was necessary previously to repeat our basic experiment, i. e. to test the substance from different zones of the anterior lobe of animals already investigated on tadpoles of the new amphibian species.

Similar to our previous experiments, the tadpoles were taken directly from nature, and classed according to their size and developmental stage. Larvae were used at various stages, long before the setting in of natural metamorphosis (hind extremities at the stage of undifferentiated bud). Every experiment involved three sets consisting of 15—20 individuals each. The tadpoles of one set served as controls and have not been subjected to any treatment; to the larvae of the other two series we implanted into the abdominal cavity pieces (1—1.5 mg) of the anterior lobe of ox hypophyses taken from the basophilic or eosinophilic zone, respectively. For the implantation fresh material was used, taken directly from the sloughter-house. The tadpoles were kept in glass vessels, water and food being renewed every second day.

7—8 days upon the beginning of the experiments all the tadpoles were killed at once. In order to evaluate the biological effect, the tadpoles and their separate organs were weighed and measured. The mean data for each indicator were then calculated, and the difference in the effect between experiment and control found in per cent to control.

Figures expressing the results of two preliminary experimental runs, are given in Table 1.

Table 1

Development of Tadpoles Implanted Substance from Basophilic or Eosinophilic Zone of the Anterior Lobe of Ox Hypophysis

	Run	Body		Tail		Intestine	
		mg	%	mg	%	mg	%
Rana ridibunda	Control	418	—	78	—	151	—
	Eosinophilic zone	415	0.7	69	11.5	144	4.6
	Basophilic zone	124	28.3	27	65.4	49	66.8
Bufo viridis	Control	201	—	34	—	99	—
	Eosinophilic zone	223	−10.9	37	−8.9	87	12.1
	Basophilic zone	122	34.3	19	44.1	52	47.3

As may be seen from Table 1, the difference in the morphogenic effect is quite distinctly expressed. The substance of the basophilic zone brings about a premature accelerated development of the tadpoles. The resorption of the larval organs within the short period of the tests proceeded very intensively.

The tadpoles of *Rana ridibunda* displayed a higher degree of the morphogenic effect. The thyrotropic action of basophilic cells has thus once more been brought out with satisfactory clearness. The substance of the eosinophilic zone failed to show a similar action. On the contrary, one could even note a certain stimulation of the growth processes in larvae of *Bufo viridis*; however, on account of the short duration of the experiment, the effect was but very slightly expressed. Consequently, we have obtained under new conditions and on new test-objects a full confirmation of the typical morphogenic action of substance from two different zones of the anterior lobe of the pituitary gland, revealed in our previous work.

In the subsequent, fundamental part of our work, we subjected to histo-physiological examination 22 pituitary glands from human subjects of various age ranging from 20 to 50 years. The material was taken from healthy subjects that fell victims of accidents, no later than 24 hours after their death. Upon removing the surrounding tissues, the pituitary gland was cut in the sagittal direction. From

Schematic drawing of sagittal sections of four human pituitary glands. *Pa*, anterior lobe; *Pp*, posterior lobe; *Pi*, intermediate lobe; *Bz*, basophilic zone; *Ez*, eosinophilic zone

the isolated anterior lobe small pieces of the basophilic zone were excised with the aid of a thin scalpel. The basophilic zone could be clearly distinguished from the grey-orange surrounding tissue by its deeper coloration and more compact consistency.

The preparation of bits for the implantation of substance from the eosinophilic zone presented no special difficulties. Part of the material was fixed and subsequently subjected to histological treatment. The difference in the coloration of the various parts of the anterior lobe can be noticed already during its morphological examination. One soon becomes aware of the fact that situation, shape and size of the basophilic zone are subject to considerable variations. This observation was supported by microscopical examination of sagittal sections made across the anterior lobe. Hence it becomes clear why the isolation of bits from the basophilic zone involved certain difficulty, for in each separate case, previous to the preparation it was necessary to become thoroughly acquainted with the surface of the section, while before separation into zones was met with no difficulties, and used to be practised mechanically. The variability of the topography of the different zones of the anterior lobe of human pituitary body is schematically represented in the figure.

The technique of testing human pituitary glands and the treatment of the results obtained were the same as described above. All our experiments were repeated many times, and the effect obtained was qualitatively unique; its quantitative expression fluctuated, however, within a rather wide range, which should be put

down to a different nicety in isolating separate glands into zones for implantation.

Tables 2 and 3 contain the results of two typical experimental series run on tadpoles of different species.

Table 2

Morphogenic Effect of Substance from Different Zones of the Anterior Lobe of Human Pituitary Gland upon Larvae of *Rana ridibunda*

Run	Body		Tail		Intestine	
	mg	%	mg	%	mg	%
Control	720	—	195	—	242	—
Eosinophilic zone	790	−9.7	198	1.5	138	43.0
Basophilic zone	252	65.0	154	21.5	26	89.3
Control	653	—	162	—	195	—
Eosinophilic zone	492	24.6	116	28.4	68	65.1
Basophilic zone	470	28.0	102	37.0	42	78.4

Table 3

Morphogenic Effect of Substance from Different Zones of the Anterior Lobe of Human Pituitary Gland upon Larvae of *Bufo viridis*

Run	Body		Tail		Intestine	
	mg	%	mg	%	mg	%
Control	229	—	36	—	104	—
Eosinophilic zone	260	−13.6	35	2.7	35	18.3
Basophilic zone	161	29.7	23	36.1	32	69.2
Control	295	—	52	—	128	—
Eosinophilic zone	253	−14.2	43	17.3	74	42.1
Basophilic zone	221	25.1	38	27.0	55	57.0

From a comparison of the data of the two tables it may be seen that, irrespective of the species of the test-object, the effect obtained is invariably the same. When the substance from the basophilic zone has been implanted, an accelerated transformation of larvae is observed. Both size and weight of the tadpole's body sharply decrease within a short period of time. The intestine shortens appreciably (down to 80—90 per cent of its original length). The tail undergoes considerable resorption.

The metamorphic alterations proceeding in the rest of organs are no less saliant. In the case substance from the eosinophilic zone be implanted, there is either no effect upon metamorphosis, or in some cases a slighter effect than with tissues from the basophilic zone is observed. This slight morphogenic effect of the substance from the eosinophilic zone should be accounted for by an admixture of basophilic elements, which undoubtedly took place in many cases because of the difficulties involved in the preparation of the relatively small anterior lobe of a human pituitary gland. The presence of basophilic cells in the eosinophilic zone proper might also have told upon the biological effect.

Data bearing on the microscopic structure and the results of the biological tests show concordantly that there is no regular distribution of the basic secretory elements in the anterior lobe of human pituitary body. The basophilic cells are concentrated mainly in a narrow region occupying a central position, or else in a region encircling that part of the eosinophilic zone which is situated in the centre.

In the basophilic cells the thyrotropic principle is localized, which is endowed with a morphogenic activity not below that of the animals already investigated.

The material at our disposal is far from being sufficient to draw any comparative histo-physiological conclusions. Yet, considerable variations in the topography, shape and size of the basophilic zone may already be pointed out, which distinguish the pituitary body of man from this organ in the animals previously investigated.

Neither does the material collected permit of any conclusions as to the variation in the relation between the zones and their biological properties in human subjects of a different age. Of undisputable interest would be a further investigation of topographic changes in the anterior lobe in human subjects of different sex, as occurring in the course of various physiological processes associated with changes in the morphogenic properties of the pituitary body.

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HISTOLOGY

CELL CHANGES IN CONNECTIVE TISSUE DURING REGENERATION. II

By T. M. YAKOVLEVA

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In a previous note ([1]) I described the changes occurring in the cells of the caudal connective tissue of the axolotl throughout the regeneration process.

The purpose of the present work was to answer the question whether, as stated by some authors, the cells forming the fin part of the regeneration blasteme («regeneration cells», according to Ide Rozas, 1936), are really caused by a gradual transformation of the fibroblasts of the fin's connective tissue. The principal signs of this transformation I see (as in my earlier note) in the changes of the relative amount of cytoplasmic pentose-nucleic acid compounds, their coacervation capacity in the shape of granules during the neutral-red reaction (formation of crinome) and in the changes of the outlines of the cell body. For these changes I have been looking on the frontal section of the caudal fin lying parallel to the chorda of each axolotl, in the direction from the wound surface towards the regions situated deep within the tissue of the fin.

At early regeneration stages, when the cytoplasm of the cells lying near the wound surface is capable of complete coacervation of the pentose-nucleic acid compounds in the shape of granules (crinome), the number of crinome granules could be counted in every cell. A total of twenty five sections were examined, and the granules were counted in 674 cells. Such a count gave an idea of the relative content of cytoplasmic pentose-nucleic acid compounds, although it involved a certain inaccuracy due to the difference in the size of granules (the cells situated near the wound surface usually contained a great number of large granules). Beginning with the boundary between the wound surface and the covering epithelium, the section was divided into areas one tenth of a millimeter large. The granules met within a given area were counted in all the fibroblasts. During a thorough examination of sections from the first axolotl investigated (No. 23, fixed 4 days upon tail amputation) I found five successive regions 0.6 mm long. These regions differed by the number of crinome granules per cell and by the number of cells containing many granules.

According to the number of granules in the cells, variation curves were plotted for every region (Table 1 and Fig. 1). Because of the similarity of the curves of the two last regions, the latter have been united into a double region.

Curve I (from 0.1 to 0.6 mm) shows that the area of the preparation immediately near the wound surface contains a great number of cells rich in pentose-nucleic acid compounds, and a great number of granules in each cell ($M = 28.89$ granules).

The great variability of the series in this region should be emphasized ($\sigma = \pm 19.11$ granules): along with cells containing 90 crinome granules, cells are met containing only nine granules.

Table 1

Regions	Distance from boundary with epithelium in mm	$M \pm m$ in granules	σ in granules	R
I	0.1—0.6	28.89 ± 1.41	± 19.11	8.66
II	0.7—1.2	13.56 ± 1.08	± 11.16	2.88
III	1.3—1.8	9.17 ± 1.07	± 9.46	2.71
IV	1.9—3.0	5.98 ± 0.49	± 4.25	15.48

Curve II (from 0.7 to 1.2 mm) is on the whole similar to curve I, but shows a considerably smaller number of cells rich in pentose-nucleic acid compounds, whereas the number of cells poor in these compounds is much greater. The average number of granules per cell is much lower ($M = 13.56$ granules). The difference between curves I and II is significant ($R = 8.66$), which may point to the essential changes taking place at the early regeneration stage in the properties of cells near the wound surface, as compared with the more remote cells.

Curve III (1.3—1.8 mm). The peak of this curve coincides with that of curve II

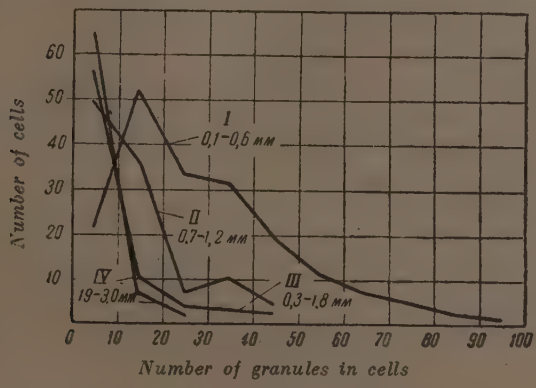

Fig. 1

(second region); the slope of the two curves is very much similar, but there is no double peak as in the two first curves. The decrease in the number of cells rich in pentose-nucleic acid compounds at the expense of those poor in them is still greater.

The average number of granules per cell is smaller ($M = 9.17$ granules). However, the cells are more homogeneous as regards the content of pentose-nucleic acid compounds in their cytoplasm ($\sigma = \pm 9.46$ granules). The difference between curves II and III is almost significant ($R = 2.88$).

Curve IV (1.9—3.0 mm) shows no cells containing large amounts of pentose-nucleic acid compounds. The variability of the cells, as regards the content of pentose-nucleic acid compounds in their cytoplasm, is very low ($\sigma = \pm 4.25$ granules). The difference between curves III and IV is almost significant ($R = 2.71$).

The gradual changes of the curve manifest in the shift of the curves to the left and in the general decrease in the range of variation, should be taken to mean that the properties of the cell of the connective tissue of the fin change gradually in the direction from the wound surface towards the more remote areas. This change consists in a gradual reduction of the number of cells containing large amounts of pentose-nucleic acid compounds, and in a diminution of the relative content of

pentose-nucleic acid compounds in the cytoplasm of one cell beginning with the wound surface and leading into the depths of the tissue. The absolute significance of the difference between the curves of the first and second regions, and, in particular, between those of the first and fourth regions ($R = 15.48$) makes it possible to speak of a probable deep difference between the cells of the first region and those of all the other regions.

If the picture be examined in a reverse order, viz. from the regions lying in the depths of the tissue and up to the wound surface, the non-homogeneity of the cells as regards the content of pentose-nucleic acid compounds in the cytoplasm arises and increases with the approach to the wound surface. This fact may be explained on the assumption that the connective tissue of the fin in an axolotl represents a tissue with an incomplete histogenesis, and contains all the transition forms, from a cell retaining its full transformation capacities, up to the condition of a mature

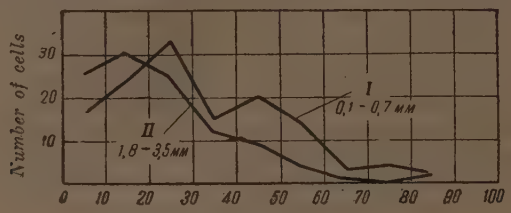

Fig. 2

fibrocyte (incapable of transformation). Those forms react in a different way to the stimuli causing their transformation into cells of the regeneration blasteme, and form different amounts of pentose-nucleic acid compounds. The accomplished fibrocytes incapable of transforming into «regeneration cells» which are situated near the wound surface at the time of amputation, remain intact and contain the same amount of pentose-nucleic acid compounds which they would have contained in the depths of the tissue.

In axolotl No. 25 fixed nine days upon tail amputation, 13 frontal sections were examined, and the granules were counted in 235 cells. Each section was divided into two regions: from 0 to 1.7 mm, and from 1.8 to 3.5 mm. Table 2 and Fig. 2 go to confirm the same regularities as brought out by the curves illustrating the material of axolotl No. 23.

Table 2

Regions	Distance from boundary with epithelium in mm	$M \pm m$ in granules	σ in granules	R
I	0.1—1.7	30.40±1.61	±18.50	3.09
II	1.8—3.5	23.33±1.62	±16.50	—

The axolotls fixed at the inflammation stage (e. g. No. 26, 11 days after amputation, and No. 28, 15 days after amputation) show [as already described in note (1)] a total lack of coacervation of the cytoplasmatic pentose-nucleic acid compounds in the shape of granules during vital staining with neutral-red. In such cases it was impossible to count the granules, but accurate drawings of the cells of the connective tissue of the fin were made in the same direction as the counts of granules at earlier regeneration stages. It was found that in region 0—0.6 mm the cells stained a blue of different intensity showing a total absence of crinome granules. As pointed out in the foregoing note, the cytoplasmatic part of these cells shows short protuberances. The cells situated at the end of the region show against a background of blue plasma the formation of first crinome granules (incomplete segregation, according to Kedrowsky). In region 0.7—1.2 mm blue granules against a light-blue basophilic background are visible, and it is only towards the end of the region.

that cells with crinome granules against a colourless background are observable (complete segregation, after Kedrowsky). In region 1.3 mm and in regions still more remote from the wound surface, all the cells contain blue granules on a completely colourless background (Fig. 3). Their cytoplasmatic part shows very long protuberances.

Therefore, throughout section for any period of the regeneration process, the same changes of coacervation capacity of the pentose-nucleic acid compounds during reactions for neutral-red may be traced, as it has been established for the entire regeneration process. The fibroblasts of regions remote from the wound surface have a long branch-like cytoplasm. The pentose-nucleic acid compounds of their cytoplasm are capable of complete coacervation in the shape of granules (crinome) during the reaction for neutral-red. The nearer to the wound surface, the more this capacity decreases, the protuberances of the cells become shorter; finally, near the wound surface only cells with short protuberances are met with a basophilic plasma entirely devoid of granules, morphologically and by the condition of their pentose-nucleic acid compounds similar to the cells of an embryo ([1]).

As a result of this work the gradual transformation of fibroblasts of the fin's connective tissue into cells of the regeneration blasteme may be considered safely established.

As a proof of this transformation serves the gradual shortening of the plasmatic protuberances of the cells, the gradual increase of the amount of pentose-nucleic acid compounds in their cytoplasm and the gradual changes of these compounds, manifested in the loss of the coacervation capacity in the shape of granules during the neutral-red reaction (the problem of the gradual changes of the nucleus and of the nucleoli has not been investigated in this work). The picture of the changes in the connective tissue of the fin of every axolotl is clearly visible both throughout the whole regeneration process [according to regeneration stages ([1])] and at every separate moment, beginning with the wound surface till regions lying deep within the tissue.

Fig. 3. Cells of axolotl No. 26. Distances between the cells and epithelium: *a*, 0.22 mm; *b*, 0.9 mm; *c*, 1.5 mm

The increase in the amount of the pentose-nucleic acid compounds at the first regeneration stages may be considered a condition responsible for their subsequent changes throughout the regeneration process. The cells of the regeneration blasteme, formed as a result of gradual changes, become both morphologically and by the state of the pentose-nucleic acid compounds in their cytoplasm, similar to the mesenchyme cells of an embryo. The absolute significance of the difference between the curves of the first and the next regions shows that these cells are endowed with entirely new properties as compared to the fibroblasts to which they owe their origin.

The fact that the changes proceed gradually means that the influence is spreading from the wound surface into the depths of the fin tissue. And a factor that requires such a considerable period of time to spread can be nothing else than a factor of chemical nature.

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PALAEONTOLOGY

A NEW RESTORATION OF *DREPANASPIS*

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(Communicated by A. A. Borissiak, Member of the Academy, 23. V. 1943)

Drepanaspis gemündenensis Schlüter is a ray-like fish of the fam. *Psammosteidae*, order *Psammosteiformes*, subcl. *Heterostraci*, cl. *Pteraspidomorphi*. It is found in the Lower Devonian Hunsrückschiefer of Rhineland, and is wide-known. It is figured nearly in all text-books of Palaeontology. However, up to the present time only the old restoration by Traquair (1, 2) is reproduced, figuring the dorsal and ventral aspects of the fish. Besides Traquair's restoration, there exist a new restoration by Patten (3) quite distinct from it, a drawing of the ventral side by Stetson (4), and some drawings by different authors, mostly repeating Traquair's restoration with trifling alterations.

In studying the Psammosteids from the Middle and Upper Devonian of the Soviet Union, I had for sake of comparison to examine the structure of *Drepanaspis*. This examination showed that the above-cited restorations, in many respects contradicting one another, swerve considerably from reality, too, even inasmuch as this latter may be ascertained from the existing material. The perusal of the published data, as well as the study of 4 specimens in the Mining Museum, Leningrad, allowed to give a new restoration, drawn by N. A. Yanshinov after my sketches and under my direction. A plaster model of *Drepanaspis* was sculptured by J. M. Eglon.

I shall touch here only upon the major traits distinguishing our restoration from the preceding ones, postponing a detailed description of *Drepanaspis* till my monograph of the *Psammosteidae*.

In the general outline of the trunk our restoration does not differ substantially from Traquair's drawings. Patten, however, figured the trunk too elongated and angular, basing, as it seems, on strongly deformed specimens.

The tail is hypocercal, with a long narrow lower lobe and a short broad upper one. In Patten's restoration it is epicercal, in Stetson's drawing, homocercal. Yet, Traquair [(2), 1905] has already shown that the longer lobe pertained to that side which he considered to be dorsal, but which, as we now know after the works by Dean (5) and Kiaer (6), is ventral. Upon the form of the tail we can conclude from Traquair's pl. II, fig. 1 [(2), 1905]. It is very like that of *Pteraspis*.

The mouth is shifted on to the dorsal side. This was the main reason why Traquair mistook this side for the ventral one. The mouth was placed on the same surface also by the subsequent writers, except Woodward (7) and Stensiö (8). These latter closed the mouth by joining the oral plates to the rostral, removing it thus from the dorsal side. They did not mention, however, by which plates in this case would the mouth be bordered on the ventral side. Yet, its position on the dorsal side in all specimens, disregarding their mode of deformation, leaves no doubt that this position is a life time, and not a post-mortem one.

In Patten's restoration there are depicted nasal openings near the upper margin of the mouth on each side of the rostral, a pineal opening behind the latter plate in the notch of the dorsal, and an anal opening behind the ventral. We do not insert all these openings in our restoration, since we did not succeed to find them in the material at hand.

The following peculiarities of our restoration as regards the form and position of individual plates should be mentioned.

In Traquair's drawing and in all its subsequent modifications, the rostral lies closely to the dorsal, fitting into the anterior notch of the latter. However, between

Fig. 1. Restoration of *Drepanaspis gemündenensis*, dorsal side, $^1/_2$.

Fig. 2. Restoration of *Drepanaspis gemündenensis*, ventral side, $^1/_2$.

them small plates are usually found and the borders of both larger plates are curved differently. That is why I follow Patten and Heintz in inserting here a zone of small plates. This results also from the homology with *Pteraspis*, in which genus, as it has been found lately, there is a belt of pineal-orbital plates between the rostral and dorsal.

As to the form of the dorsal, our restoration takes an intermediate position between those of Traquair and Patten. In the former author's figure, the dorsal is too wide, in the latter's, too narrow. Traquair's drawing does not show the hind notch and the ridge formed in the hind half of the plate by the underlying bases of anterior ridge scales.

The ventral is restored by us quite differently than by all previous authors. Its anterior end is not narrow and rounded, as in the figures by Traquair and Pat-

ten, but broad and slightly concave. Only in Stetson's drawing the front margin is direct, but not long enough. Yet the Mining Museum specimens show clearly its true form. All authors depict only a quite small notch at the hind end of the ventral, but in our material, as well as in Traquair's photographs, there is clearly discernible a narrow deep notch, occupying about $^1/_3$ of the length of the ventral and covered with a separate wedge-shaped plate. I have already mentioned this notch (°). It is of great taxonomic value, as it reappears in several Psammosteids: *Schizosteus*, *Tartuosteus* n. g., and *Pycnosteus*.

As regards the lateral plates, i. e. the branchial and the cornual, our drawing is nearer that of Traquair. Patten figured the former plate too broad at the hind end, the latter protruding from the posterior margin of the trunk. This latter position must be due to a post-mortem displacement.

Essential new points are added by our restoration to the interpretation of the plates at the front margin and the anterior of the lateral margins of the armour. Anteriorly of the branchial lies the postorbital, embracing dorsally and ventrally the pointed anterior end of the former. The ventral wing of the postorbital was overlooked by all preceding authors. Yet, the character of the suture between the branchial and the postorbital is essential for the homology of the latter plate. It is generally maintained that the postorbital is a result of the subdivision into two parts of the orbital plate of the pteraspidian ancestors of *Drepanaspis* (Stensiö, Heintz). However, according to Gross, in *Pteraspis dunensis* the branchial consists usually of two parts, the hind one fitting into a re-entrant angle of the front one in the same way as the branchial fits into the postorbital in *Drepanaspis*. The latter plate therefore is homologous with the anterior part of the branchial of *Pteraspis*, and does not represent a product of subdivision of the orbital.

The orbital is triangular in its dorsal part, as in Patten's figure, while in Traquair's restoration it is trapezoidal. Yet, its triangular form is clearly seen in many photographs. It also bends on to the ventral side, which is not shown in previous restorations.

Further anteriorly follow the oral plates. The lateral pair of the orals close the corners of the mouth, bordering from the front on the orbitals. The following 5 plates (two pairs and one unpaired) form the lower border of the mouth. These plates are rather large and lie on the ventral side longitudinally. The median one of them is cleft anteriorly. All other restorations figure these plates in a different way, but each of them incorrectly. Traquair and the subsequent authors insert here 3 pairs of small plates. Patten, 5 pairs of small plates, freely protruding forward and bordering on a pair of larger plates of the ventral side. In a softened form this is repeated by Heintz. However, careful examination of the Mining Museum specimens and of the photographs published by Traquair and Stetson shows that the latter author's restoration is nearest the truth. It differs from the present one only in the lateral pair of the orals being directed radially, not longitudinally. Thus, in *Drepanaspis*, as compared with *Pteraspis*, a confluence of the numerous (12—16) orals of the latter genus is observed.

The scales of the tail are partly very large and deep. These large scales are clearly seen in most specimens, but figured only in Patten's drawing. As regards the form and number of the small plates, our restoration is nearer Traquair's than Patten's one.

The homology of the individual plates of *Drepanaspis* and *Pteraspis* is now universally adopted. I introduce new points only in the interpretation of the «postorbital» which it is better to designate the prebranchial.

The main feature distinguishing *Drepanaspis* from the Pteraspids is the presence of zones of small plates dividing the large plates homologous with the plates of *Pteraspis*. This is usually interpreted (Stensiö, Heintz, Brotzen) as a proof of an advancing subdivision of the armour in *Heterostraci*. However, the small plates appear in *Drepanaspis* where in *Pteraspis* are situated the sensory canals (lateral, ventral, postpineal, supraorbital) which are incorporated into the large plates only at later stages of growth. Thus, in *Pteraspis* these zones along the canals were left unprotected by the armour in the young. In *Drepanaspis*, on the contrary, these

zones became covered with small plates; the growth of the large plates ended without having attained these canals. Therefore, we have here to deal not with a subdivision, but with a neoformation useful in two respects: 1) the young had no vulnerable spaces, unprotected by the armour, and 2) the growth of the armour was facilitated. Owing to this, the descendants of *Drepanaspis*, the Psammosteids, flourished during the Middle and Upper Devonian when the Pteraspids had already died out, and greatly outpassed these in size: some of them attained gigantic dimensions.

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